education

DEPARTMENT: EDUCATION
MPUMALANGA PROVINCE

Grade 12

Supplementary Study Material

Mathematics

Mathematic Paper 01
November 2009

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Together Educating the Nation

QUESTION 1

1.1 Solve for x:

1.1.1 $x(x-1)=30$ (3)

1.1.2 $3x^2-5x+1=0$ (Correct to ONE decimal place) (4)

1.1.3 $15x-4<9x^2$ (4)

1.2 Solve simultaneously for x and y in the following set of equations:

$x-y=3$
$x^2-xy-2y^2-7=0$ (5)

1.3 Calculate the exact value of:

$$\frac{\sqrt{10^{2009}}}{\sqrt{10^{2011}}-\sqrt{10^{2007}}}$$ (Show ALL calculations.) (3)

1.4 Simplify completely without the use of a calculator:

$$\left(1+\sqrt{2x^2}\right)^2-\sqrt{8x^2}$$ (3)
[22]

QUESTION 2

2.1 Tebogo and Matthew's teacher has asked that they use their own rule to construct a sequence of numbers, starting with 5. The sequences that they have constructed are given below.

Matthew's sequence: 5 ; 9 ; 13 ; 17 ; 21 ; ...
Tebogo's sequence: 5 ; 125 ; 3 125 ; 78 125 ; 1 953 125 ; ...

Write down the n^{th} term (or the rule in terms of n) of:

2.1.1 Matthew's sequence (3)

2.1.2 Tebogo's sequence (2)

2.2 Nomsa generates a sequence which is both arithmetic and geometric. The first term is 1. She claims that there is only one such sequence. Is that correct? Show ALL your workings to justify your answer. (5)
[10]

QUESTION 3

Given: $\sum_{t=0}^{99}(3t-1)$

3.1 Write down the first THREE terms of the series. (1)

3.2 Calculate the sum of the series. (4)
[5]

QUESTION 4

The following sequence of numbers forms a quadratic sequence:

$-3\ ;-2\ ;-3\ ;-6\ ;-11\ ;\ldots$

4.1 The first differences of the above sequence also form a sequence. Determine an expression for the general term of the first differences. (3)

4.2 Calculate the first difference between the 35^{th} and 36^{th} terms of the quadratic sequence. (2)

4.3 Determine an expression for the n^{th} term of the quadratic sequence. (4)

4.4 Explain why the sequence of numbers will never contain a positive term. (2)
[11]

QUESTION 5

Data regarding the growth of a certain tree has shown that the tree grows to a height of 150 cm after one year. The data further reveals that during the next year, the height increases by 18 cm. In each successive year, the height increases by $\frac{8}{9}$ of the previous year's increase in height. The table below is a summary of the growth of the tree up to the end of the fourth year.

	First year	Second year	Third year	Fourth year
Tree height (cm)	150	168	184	$198\frac{2}{9}$
Growth (cm)		18	16	$14\frac{2}{9}$

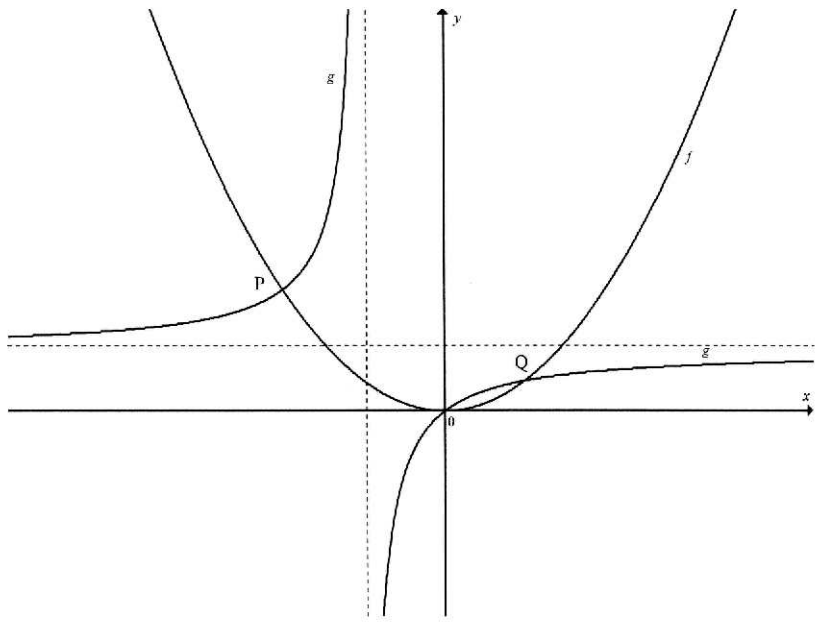
5.1 Determine the increase in the height of the tree during the seventeenth year. (2)

5.2 Calculate the height of the tree after 10 years. (3)

5.3 Show that the tree will never reach a height of more than 312 cm. (3)
[8]

QUESTION 6

Sketched below are the graphs of $f(x)=\frac{1}{2}x^2$ and $g(x)=-\frac{1}{x+1}+1$.
P and Q are the points of intersection of f and g.



6.1 Show that the coordinates of P and Q are P$(-2\ ;2)$ and Q$(1\ ;\frac{1}{2})$ respectively. (6)

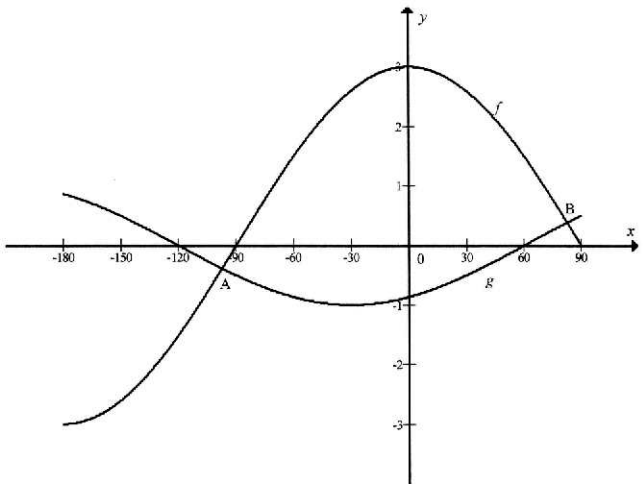
6.2 An axis of symmetry of the graph of g is a straight line defined as $y=mx+c$, where $m>0$. Write down the equation of this straight line in the form $y=h(x)=\ldots$ (2)

6.3 Determine the equation of h^{-1} in the form $y=\ldots$ (2)

6.4 Show algebraically that $g(x)+g\left(\frac{1}{x}\right)=g(-x).g(x-1)$. $(x\neq 0$ or $x\neq 1)$ (3)
[13]

QUESTION 7

The graphs of $f(x)=3\cos x$ and $g(x)=\sin(x-60°)$ are sketched below for $x\in[-180°;90°]$.



7.1 Write down the range of f. (1)

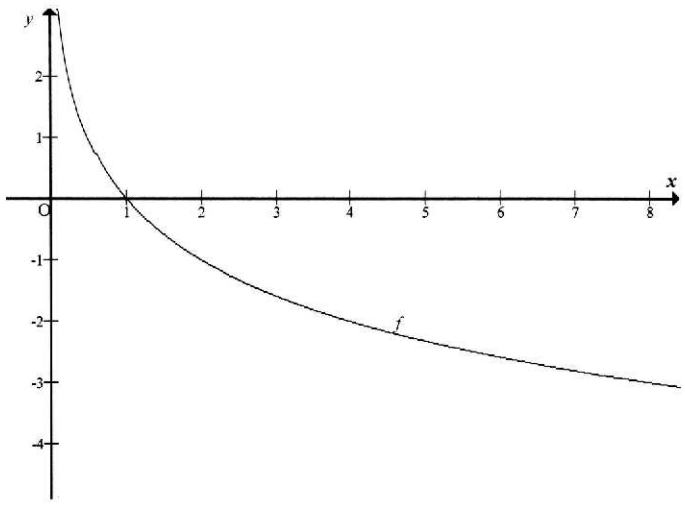
7.2 If A$(-97{,}37°\ ;-0{,}38)$, write down the coordinates of B. (3)

7.3 Write down the period of $g(3x)$. (2)

7.4 Write down a value of x for which $g(x)-f(x)$ is a maximum. (2)
[8]

QUESTION 8

Sketched below is the graph of $f(x) = -\log_2 x$.



8.1 Write down the domain of f. (1)

8.2 Write down the equation of f^{-1} in the form $y = \ldots$ (1)

8.3 Write down the equation of the asymptote of f^{-1}. (1)

8.4 Explain how, using the graph of f, you would sketch the graphs of:

8.4.1 $g(x) = \log_2 x$ (1)

8.4.2 $h(x) = 2^{-x} - 5$ (3)

8.5 Use the graph of f to solve for x where $\log_2 x < 3$. (3)
[10]

QUESTION 9

9.1 A photocopier valued at R24 000 depreciates at a rate of 18% p.a. on the reducing-balance method. After how many years will its value be R15 000? (4)

9.2 A car that costs R130 000 is advertised in the following way: 'No deposit necessary and first payment due three months after date of purchase.' The interest rate quoted is 18% p.a. compounded monthly.

9.2.1 Calculate the amount owing two months after the purchase date, which is one month before the first monthly payment is due. (3)

9.2.2 Herschel bought this car on 1 March 2009 and made his first payment on 1 June 2009. Thereafter he made another 53 equal payments on the first day of each month.

(a) Calculate his monthly repayments. (3)

(b) Calculate the total of all Herschel's repayments. (1)

9.2.3 Hashim also bought a car for R130 000. He also took out a loan for R130 000, at an interest rate of 18% p.a. compounded monthly. He also made 54 equal payments. However, he started payments one month after the purchase of the car. Calculate the total of all Hashim's repayments. (4)

9.2.4 Calculate the difference between Herschel's and Hashim's total repayments. (1)
[16]

QUESTION 10

10.1 Differentiate $f(x)$ from first principles if $f(x) = -2x^2 + 3$. (5)

10.2 Evaluate: $\frac{dy}{dx}$ if $y = x^2 - \frac{1}{2x^3}$ (2)
[7]

QUESTION 11

Given: $f(x) = -x^3 + x^2 + 8x - 12$

11.1 Calculate the x-intercepts of the graph of f. (5)

11.2 Calculate the coordinates of the turning points of the graph of f. (5)

11.3 Sketch the graph of f, showing clearly all the intercepts with the axes and turning points. (3)

11.4 Write down the x-coordinate of the point of inflection of f. (2)

11.5 Write down the coordinates of the turning points of $h(x) = f(x) - 3$. (2)
[17]

QUESTION 12

A tourist travels in a car over a mountainous pass during his trip. The height above sea level of the car, after t minutes, is given as $s(t) = 5t^3 - 65t^2 + 200t + 100$ metres. The journey lasts 8 minutes.

12.1 How high is the car above sea level when it starts its journey on the mountainous pass? (2)

12.2 Calculate the car's rate of change of height above sea level with respect to time, 4 minutes after starting the journey on the mountainous pass. (3)

12.3 Interpret your answer to QUESTION 12.2. (2)

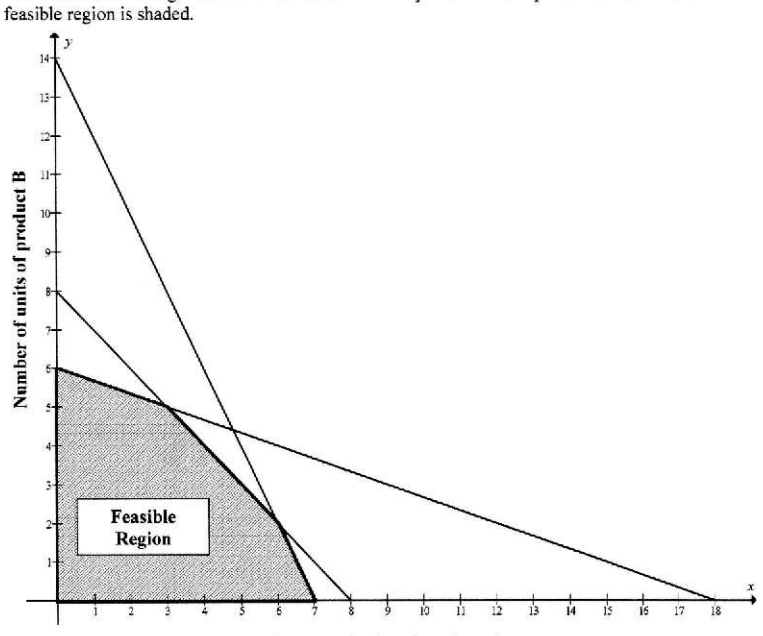
12.4 How many minutes after the journey has started will the rate of change of height with respect to time be a minimum? (3)
[10]

QUESTION 13

A steel manufacturer makes two kinds of products, product A and B, having parts that must be cut, assembled and finished. The manufacturer is aware that it can sell as many products as it can produce.

Let x and y be the number of units of product A and product B that are manufactured every day respectively.

The constraints that govern the manufacture of the products are represented below and the feasible region is shaded.



13.1 Write down the constraints in terms of x and y that represent the above information. (7)

13.2 If product A yields a profit of R30 per item and product B yields R40 per item, write down the equation indicating the daily profit in terms of x and y. (2)

13.3 Determine the number of units of product A and product B that the manufacturer needs to produce in order to maximise his daily profit. A diagram is provided on DIAGRAM SHEET 1. (2)

13.4 The manufacturer would like the maximum profit to be at (6 ; 2) for the profit equation P = mx + c. Determine the values of m which will satisfy this condition (2)
[13]

TOTAL: 150

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INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\sum_{i=1}^{n} (a+(i-1)d) = \frac{n}{2}(2a+(n-1)d)$

$\sum_{i=1}^{n} ar^{i-1} = \frac{a(r^n-1)}{r-1}$; $r \neq 1$ $\sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC:

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha+\beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha-\beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha-\beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum (x-\bar{x})(y-\bar{y})}{\sum (x-\bar{x})^2}$

Memorandum

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 1

1.1.1

$x(x-1) = 30$
$x^2 - x = 30$
$x^2 - x - 30 = 0$
$(x-6)(x-5) = 0$
$x = 6$ or $x = -5$

If implied as equation : No penalty

If there is no equals sign or the equation is not = 0: No penalty

If $x = 6$ is answer by inspection : 1 / 3

Both correct answers no calculation : 1 / 3

✓ simplification (multiplying out brackets)
✓ factors
✓ both answers (3)

OR

$x(x-1) = 30$
$x^2 - x = 30$
$x^2 - x - 30 = 0$
$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-30)}}{2(1)}$
$= \frac{1 \pm \sqrt{121}}{2}$
$= \frac{1 \pm 11}{2}$
$x = 6$ or $x = -5$

✓ simplification (multiplying out brackets)
✓ substitution into formula
✓ both answers *(ca)* (3)

1.1.2

$3x^2 - 5x + 1 = 0$
$a = 3 \quad b = -5 \quad c = 1$
$x = \frac{-(-5) \pm \sqrt{25 - 4(3)(1)}}{2(3)}$
$= \frac{5 \pm \sqrt{13}}{6}$
$x = 1{,}4$ or $x = 0{,}2$

NOTE:
Penalty 1 for incorrect rounding off in either answer

Using calculator incorrectly: Max: 2 / 4
Answers will be $x = 5{,}6$ or $4{,}4$

Incorrect formula: max 1 / 4

If $x = \frac{5 \pm \sqrt{37}}{6}$ then CA applies
$x = 1{,}8$ and $-0{,}2$: Max 3 / 4

Correct answer only: 2 / 4

If factorising: 0 / 4

If $x = \frac{5 \pm \sqrt{13}}{6}$ only, then 2 / 4

If $x = 5 \pm \frac{\sqrt{13}}{6}$ only, then 1 / 4

✓ substitution into correct formula
✓ $\sqrt{13}$
✓✓ values of x *(CA with formula)* (4)

OR

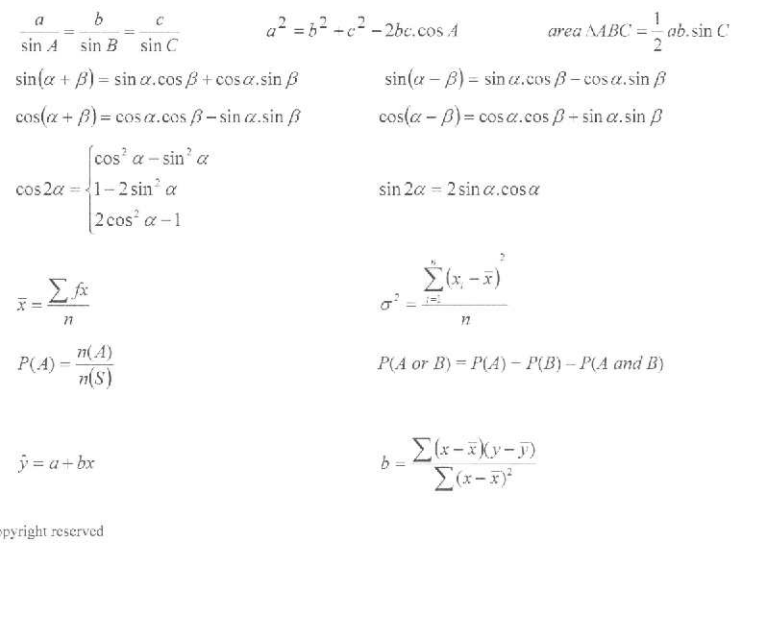
 Please turn over

CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 1

QUESTION 13.3



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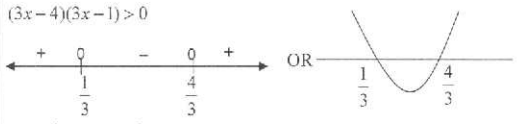
1.1.2 contd

$3x^2 - 5x + 1 = 0$
$x^2 - \frac{5}{3}x = -\frac{1}{3}$
$x^2 - \frac{5}{3}x + \frac{25}{36} = -\frac{1}{3} + \frac{25}{36}$
$\left(x - \frac{5}{6}\right)^2 = \frac{13}{36}$
$x - \frac{5}{6} = \frac{\pm\sqrt{13}}{6}$
$x = \frac{5 \pm \sqrt{13}}{6}$
$x = 1{,}4$ or $x = 0{,}2$

✓ correct method of completing the square
✓ $\sqrt{13}$
✓✓ values of x *(CA with formula)* (4)

1.1.3

$-9x^2 + 15x - 4 < 0$
$9x^2 - 15x + 4 > 0$
$(3x-4)(3x-1) > 0$



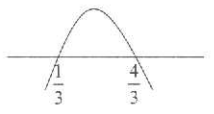
$x < \frac{1}{3}$ or $x > \frac{4}{3}$

Answer can be given as: $x \in \left(-\infty; \frac{1}{3}\right) \cup \left(\frac{4}{3}; \infty\right)$

✓ factors
✓ correct inequality sign
✓ $\frac{1}{3}$; $\frac{4}{3}$
✓ answer (4)

OR

$-9x^2 + 15x - 4 < 0$
$(-3x+4)(3x-1) < 0$
$x < \frac{1}{3}$ or $x > \frac{4}{3}$



✓ factors
✓ correct inequality sign
✓ $\frac{1}{3}$; $\frac{4}{3}$
✓ answer (4)

NOTE:

If stop at factorisation: 2 / 4

If incorrect factors: CA applies 3 / 4

If answer : $\frac{1}{3} < x < \frac{4}{3}$ then 3 / 4

If $x < \frac{1}{3}$ AND $x > \frac{4}{3}$ then 3 / 4

 Please turn over

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1.2

Substitute $x = y + 3$ in $x^2 - xy - 2y^2 - 7 = 0$

$(y+3)^2 - y(y+3) - 2y^2 - 7 = 0$ ✓ substitution

$y^2 + 6y + 9 - y^2 - 3y - 2y^2 - 7 = 0$

$2y^2 - 3y - 2 = 0$ ✓ standard form

$(2y+1)(y-2) = 0$ ✓ factors

$y = -\frac{1}{2}$ or $y = 2$ ✓ both y-values

$x = 2\frac{1}{2}$ or $x = 5$ ✓ both x-values (5)

NOTE:
If the equation is changed to a linear equation, then max 2 / 5

There are no penalties for not putting = 0.

OR

$y = x - 3$ ✓ substitution

$x^2 - x(x-3) - 2(x-3)^2 - 7 = 0$

$x^2 - x^2 + 3x - 2(x^2 - 6x + 9) - 7 = 0$

$0 = 2x^2 - 15x + 25$ ✓ standard form

$0 = (2x-5)(x-5)$ ✓ factors

$x = 2\frac{1}{2}$ or $x = 5$ ✓ both x-values

$y = -\frac{1}{2}$ or $y = 2$ ✓ both y-values (5)

1.3

$$\frac{10^{\frac{2009}{2}}}{10^{\frac{2011}{2}} - 10^{\frac{2007}{2}}}$$ ✓ convert to indices

$$= \frac{10^{\frac{2009}{2}}}{10^{\frac{2007}{2}}(100-1)}$$ ✓ common factor

$$= \frac{10}{99}$$ ✓ answer (3)

OR

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1.3 contd

$$\frac{10^{1004}\sqrt{10}}{10^{1005}\sqrt{10} - 10^{1003}\sqrt{10}}$$ ✓ convert to indices

$$= \frac{10^{1004}\sqrt{10}}{\sqrt{10}(10^{1005} - 10^{1003})}$$ ✓ common factor

$$= \frac{10^{1004}}{10^{1003}(100-1)}$$ ✓ answer (3)

$$= \frac{10}{99}$$

OR

$$\frac{\sqrt{10^{2009}}}{\sqrt{10^{2009}.10^2} - \sqrt{10^{2009}.10^{-2}}}$$ ✓ convert to indices

$$= \frac{\sqrt{10^{2009}}}{\sqrt{10^{2009}}(10 - 10^{-1})}$$ ✓ common factor

$$= \frac{1}{10 - \frac{1}{10}}$$ ✓ answer (3)

$$= \frac{1}{\frac{99}{10}}$$

$$= \frac{10}{99}$$

OR

$$\frac{\sqrt{10^{2000}}\sqrt{10^9}}{\sqrt{10^{2000}.10^{11}} - \sqrt{10^{2000}.10^7}}$$ ✓ convert to indices

$$= \frac{\sqrt{10^{2000}}\sqrt{10^9}}{\sqrt{10^{2000}}\left(\sqrt{10^{11}} - \sqrt{10^7}\right)}$$ ✓ common factor

$$= \frac{\sqrt{10^9}}{\sqrt{10^{11}} - \sqrt{10^7}}$$ ✓ answer (3)

$$= \frac{10\sqrt{10^7}}{100\sqrt{10^7} - \sqrt{10^7}}$$

$$= \frac{10\sqrt{10^7}}{\sqrt{10^7}(100-1)}$$

$$= \frac{10}{99}$$

OR

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$$\frac{\sqrt{10^{2007}}.\sqrt{10^2}}{\sqrt{10^{2007}.10^4} - \sqrt{10^{2007}}}$$ ✓ convert to indices

$$= \frac{10\sqrt{10^{2007}}}{\sqrt{10^{2007}}(\sqrt{10^4} - 1)}$$ ✓ common factor

$$= \frac{10}{100-1}$$

$$= \frac{10}{99}$$ ✓ answer (3)

OR

Let $x = 2009$

$$\frac{\sqrt{10^x}}{\sqrt{10^{x+2}} - \sqrt{10^{x-2}}}$$

$$= \frac{10^{\frac{x}{2}}}{10^{\frac{x}{2}}.10 - 10^{\frac{x}{2}}.10^{-1}}$$ ✓ convert to indices

$$= \frac{10^{\frac{x}{2}}}{10^{\frac{x}{2}}\left(10 - 10^{-1}\right)}$$ ✓ common factor

$$= \frac{1}{10 - \frac{1}{10}}$$

$$= \frac{1}{\frac{99}{10}}$$ ✓ answer (3)

$$= \frac{10}{99}$$

1.4

$$\left(1 + \sqrt{2x^2}\right)^2 - \sqrt{8x^2}$$

$$= 1 + 2\sqrt{2x^2} + 2x^2 - \sqrt{4}.\sqrt{2x^2}$$ ✓ expansion / multiplication $1 + 2\sqrt{2x^2} + 2x^2$

$$= 1 + 2\sqrt{2x^2} + 2x^2 - 2\sqrt{2x^2}$$ ✓ $\sqrt{8x^2} = 2\sqrt{2x^2}$

$$= 1 + 2x^2$$ ✓ answer (3)

OR

$$\left(1 + \sqrt{2x^2}\right)^2 - \sqrt{8x^2}$$ ✓ expansion / multiplication $1 + \sqrt{8x^2} + 2x^2$

$$= 1 + \sqrt{8x^2} + 2x^2 - \sqrt{8x^2}$$ ✓ $\sqrt{8x^2} = 2\sqrt{2x^2}$

$$= 1 + 2\sqrt{2x^2} + 2x^2 - 2\sqrt{2x^2}$$ ✓ answer (3)

$$= 1 + 2x^2$$

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1.4 contd

OR

$$\left(1 + \sqrt{2x^2}\right)^2 - \sqrt{8x^2}$$

$$= 1 + 2\sqrt{2}x + 2x^2 - 2\sqrt{2}x \quad \text{or} \quad = 1 - 2\sqrt{2}x + 2x^2 + 2\sqrt{2}x$$ ✓expansion / multiplication

$$= 1 + 2x^2$$ ✓simplification ✓ answer (3)

Note: $\sqrt{x^2} = x$ if $x > 0$ and $-x$ if $x < 0$

OR

$$\left(1 + \sqrt{2x^2}\right)^2 - \sqrt{8x^2}$$ ✓expansion / multiplication $1 + 2.(2x^2)^{\frac{1}{2}} + 2x^2$

$$= \left(1 + (2x^2)^{\frac{1}{2}}\right)^2 - 8^{\frac{1}{2}}x$$ ✓simplification

$$= 1 + 2.(2x^2)^{\frac{1}{2}} + 2x^2 - 8^{\frac{1}{2}}x$$ ✓ answer (3)

$$= 1 + 2.2^{\frac{1}{2}}x + 2x^2 - 8^{\frac{1}{2}}x$$

$$= 1 + 8^{\frac{1}{2}}x + 2x^2 - 8^{\frac{1}{2}}x$$

$$= 1 + 2x^2$$

Note: $\sqrt{x^2} = x$ if $x > 0$ and $-x$ if $x < 0$

OR

Let $2x^2 = y$

$$\left(1 + \sqrt{2x^2}\right)^2 - \sqrt{8x^2}$$

$$= (1 + \sqrt{y})^2 - \sqrt{4y}$$

$$= 1 + 2\sqrt{y} + y - 2\sqrt{y}$$

$$= 1 + y$$

$$= 1 + 2x^2$$

[22]

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QUESTION 2

2.1.1	$T_n = 4n+1$ **OR** $T_n = 5+(n-1)(4)$ $= 4n+1$ **NOTE:** If $T_n = 5+(n-1)(4)$ then full marks	✓✓✓ Answer only (3) ✓ $d=4$ ✓ substitution ✓ answer (3)
2.1.2	$T_n = 5(25)^{n-1}$	✓ $r=25$ ✓ answer (2)
2.2	The sequence is $1 ; 1+d ; 1+2d ; 1+3d ; \ldots$ (AP) and $1 ; r ; r^2 ; r^3 ; \ldots$ (GP) $\therefore 1+d = r$ and $d = r-1$ But $1+2d = r^2$ $1+2(r-1) = r^2$ $r^2-2r+1=0$ $(r-1)^2=0$ $r=1$ OR $r^2 = 1+2d$ $(1+d)^2 = 1+2d$ $1+2d+d^2 = 1+2d$ $d^2=0$ $d=0$ $r=1$ $\therefore d=0$ $\therefore$ the one and only such sequence is $1 ; 1; 1 ; \ldots$ Nomsa is correct. **OR** $T_1 = 1$ Let the sequence be $1 ; a ; b ; \ldots$ Geometric: $r = \frac{a}{1} = \frac{b}{a}$ $a^2 = b$ Arithmetic: $d = a-1 = b-a$ $2a-1=b$ $2a-1=a^2$ $0=a^2-2a+1$ $0=(a-1)^2$ $a=1$ $b=1$ Sequence is $1 ; 1 ; 1 ; \ldots$ Nomsa is correct If: Sequence is $1 ; 1 ; 1 ; 1 ; 1 ; 1 ; \ldots$ Then $d=0$ $r=1$ Therefore only one sequence exists. Nomsa is correct Max 3 / 5 If the candidate only gives Sequence is $1 ; 1 ; 1 ; 1 ; 1 ; 1 ; \ldots$ then 2 / 5 If $ar^{n-1} = a+(n-1)d$ only then 1 / 5	✓✓reason (5) ✓ Setting up sequence ✓ $a^2=b$ ✓ $b=2a-1$ ✓ $a=1$ ✓$b=1$ (5) **[10]**

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 3

3.1	$-1+2+5-\ldots$ **OR** $-1 ; 2 ; 5$	✓ all three terms (1)
3.2	$S_n = -1+2+5+8+\ldots$to 100 terms $S_n = \frac{n}{2}[2a+(n-1)d]$ $S_{100} = \frac{100}{2}[2(-1)+(100-1)(3)]$ $= 50[-2+297]$ $= 14\,750$ **OR** $S_n = -1+2+5+8+\ldots$to 100 terms $T_{100} = 3(100)-4$ $= 296$ $S_n = \frac{n}{2}[T_1+T_{100}]$ $S_{100} = \frac{100}{2}[-1+296]$ $= 50[295]$ $= 14\,750$ **NOTE:** If $S_n = -1+2+5+8+\ldots$to 99 terms $S_n = \frac{n}{2}[2a+(n-1)d]$ $S_{99} = \frac{99}{2}[2(-1)+(99-1)(3)]$ $= \frac{99}{2}[-2+294]$ $= 14454$ **Then 3 / 4** Answer only: 4 / 4 Apply consistent accuracy. This is the answer if series is $2+5+8+$ $S_n = 2+5+8+\ldots$to 100 terms $S_n = \frac{n}{2}[2a+(n-1)d]$ $S_{100} = \frac{100}{2}[2(2)+(100-1)(3)]$ $= 50[4+297]$ $= 15050$ **Then 4 / 4**	✓ formula ✓ $n=100$ ✓ substitution ✓ answer (4) **[5]**

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 4

4.1	The first differences are $1; -1; -3; -5; \ldots$. These form a linear pattern $T_n = 1+(n-1)(-2)$ $= 3-2n$ **OR** $T_n = -2n+3$ **ANSWER ONLY:** Full marks	✓ pattern ✓$d=-2$ ✓ answer (3)
4.2	Between the 35th and 36th terms of the quadratic sequence lies the 35th first difference 35th first difference $= 3-2(35)$ $= -67$ **OR** From the quadratic sequence: $P_{36} = -1158$ and $P_{35} = -1091$ 35th first difference $= -1158-(-1091)$ $= -67$ If substitute and get $T_{35} = -2(35)+3 = -67$ and $T_{36} = -2(36)+3 = -69$, leading to the answer -2 then **1 / 2**	✓ substitution of 35 into $T_n = -2n+3$ ✓ answer (2) ✓ $P_{36} = -1158$ and $P_{35} = -1091$ ✓ answer (2)
4.3	Second difference of terms is -2. $P_n = an^2+bn+c$ $a=-1$. $3a+b=1$ $-3+b=1$ $b=4$ $a+b+c=-3$ $-1+4+c=-3$ $c=-6$ $P_n = -n^2+4n-6$ **OR** If the general term has been worked out correctly in 4.2 and not redone in 4.3 but answer just written down then **4 / 4**	✓ $a=-1$ ✓ substitution ✓ $b=4$ ✓ $c=-6$ (4)

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

4.3 contd	Second difference of terms is -2. $P_n = an^2+bn+c$ $a=-1$. $P_0 = -6 = c$ $P_n = -n^2+bn-6$ $-3 = -(1)^2+(1)b-6$ $b=4$ $P_n = -n^2+4n-6$ **OR** $P_n = \frac{n-1}{2}[2(\textit{first first difference})+(n-2)(\textit{second difference})]+P_1$ $P_n = \frac{n-1}{2}[2(1)+(n-2)(-2)]-3$ $P_n = n-1-(n-2)(n-1)-3$ $P_n = n-1-n^2+3n-2-3$ $P_n = -n^2+4n-6$ **OR** $P_n = (n-1)P_2-(n-2)P_1+2nd\ difference\frac{(n-1)(n-2)}{2}$ $P_n = (n-1)(-2)-(n-2)(-3)-2\frac{(n-1)(n-2)}{2}$ $P_n = -2n+2+3n-6-n^2+3n-2$ $P_n = -n^2+4n-6$ **OR** $P_n = \frac{(n-2)(n-3)T_1-2(n-1)(n-3)T_2+(n-2)(n-1)T_3}{2}$ $P_n = \frac{(n^2-5n+6)(-3)-2(n^2-4n+3)(-2)+(n^2-3n+2)(-3)}{2}$ $P_n = \frac{-3n^2+15n-18+4n^2-16n+12-3n^2+9n-6}{2}$ $P_n = -n^2+4n-6$ **OR**	

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

4.3 contd

$P_2 - P_1 = T_1$
$P_3 - P_2 = T_2$
$P_4 - P_3 = T_3$
$P_n - P_{n-1} = T_{n-1}$
$P_n - P_1 = T_1 + T_2 + \ldots + T_{n-1}$
$P_n - P_1 = \frac{n-1}{2}[2(1) + (n-2)(-2)]$
$P_n - (-3) = (n-1)(3-n)$
$P_n = -n^2 + 4n - 6$

4.4

Maximum value of T_n is $\frac{4(-1)(-6) - 4^2}{4(-1)} = -2$ ✓ max value – 2

The maximum value is negative and hence the sequence can not have any positive terms as the function is maximum valued ✓ explanation (2)

OR

$-n^2 + 4n - 6$
$= -(n-2)^2 + 4 - 6$
$= -(n-2)^2 - 2$ ✓ max value – 2 ✓ explanation (2)

The function has a maximum-value of – 2 and therefore the pattern will never have positive values.

OR

$T_n = -n^2 + 4n - 6$
$\frac{d}{dn}(T_n) = -2n + 4$
$0 = -2n + 4$
$n = 2$
$T_2 = -(2)^2 + 4(2) - 6$
$= -2$ ✓ max value – 2 ✓ explanation (2)

The function has a maximum-value of – 2 and therefore the pattern will never have positive values.

OR

As the sequence decreases from the second term onwards and the second term is negative, the sequence will never have a positive term. ✓✓ answer (2)

OR

$T_n = -n^2 + 4n - 6$
$\frac{d}{dn}(T_n) = -2n + 4$
$\frac{d}{dn}(T_n) < 0$ for $n > 2$ and $T_2 < 0$ so the sequence decreases and stays negative ✓✓ answer (2)

[11]

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QUESTION 6

6.1

$\frac{1}{2}x^2 = -\frac{1}{x+1} + 1$ ✓ equating
$x^2(x+1) = -2 + 2(x+1)$ ✓ multiplication by LCD
$x^3 + x^2 = -2 + 2x + 2$
$x^3 + x^2 - 2x = 0$ ✓ standard form
$x(x^2 + x - 2) = 0$ ✓ common factor
$x(x+2)(x-1) = 0$ ✓ factorisation of quadratic
$x = 0$ *or* $x = -2$ *or* $x = 1$
$y = 0$ or $y = \frac{1}{2}(-2)^2$ or $y = \frac{1}{2}(1)^2$
$y = 2$ or $y = \frac{1}{2}$ ✓ *y*-answer
P(– 2 ; 2) answer P(– 2 ; 2)
$Q\left(1; \frac{1}{2}\right)$ answer $Q\left(1; \frac{1}{2}\right)$ (6)

OR

$\frac{1}{2}(-2)^2 = 2 \quad \therefore$ (– 2 ; 2) lies on $f(x) = \frac{1}{2}x^2$ ✓ substitution
$-\frac{1}{(-2)+1} + 1 = 2 \quad \therefore$ (– 2 ; 2) lies on $g(x) = -\frac{1}{x+1} + 1$ ✓ substitution
$\therefore$ (– 2 ; 2) is one of the points P, O or Q. From the graph it is P ✓ P lies on *f* and *g*
$\frac{1}{2}(1)^2 = \frac{1}{2} \quad \therefore$ (– 2 ; 2) lies on $f(x) = \frac{1}{2}x^2 \therefore \left(1; \frac{1}{2}\right)$ is one of the points P, O or Q. From the graph it is Q ✓ substitution ✓ substitution
$-\frac{1}{(1)+1} + 1 = \frac{1}{2} \quad \therefore$ Q lies on $g(x) = -\frac{1}{x+1} + 1$
$\therefore \left(1; \frac{1}{2}\right)$ is one of the points P, O or Q. From the graph it is Q ✓ Q lies on *f* and *g* (6)

6.2

For $m > 0$, $m = 1$ ✓ gradient $m = 1$
the equation of the axis of symmetry is $y = x + c$.
$1 = (-1) + c$
$c = 2$ ✓ $c = 2$
Therefore the equation is $y = h(x) = x - 2$. (2)

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QUESTION 5

5.1

First year: 150
Second year: 150 + 18 = 168
Third year: $168 - \frac{8}{9}(18) = 184$
Growth = $18\left(\frac{8}{9}\right)^{n-2}$ after *n* years ✓ general terms
17^{th} year growth is $18\left(\frac{8}{9}\right)^{17-2} = 3{,}08$ cm ✓ answer (2)

	Yr 1	Yr 2	Yr 3	Yr 4	Yr 5	Yr 6	Yr 7	Yr 8	Yr 9
Ht	150	168	184	198,2	210,84	222,07	232,06	240,94	248,83
Inc		18	16	14,2	12,64	11,23	9,99	8,88	7,89

	Yr 10	Yr 11	Yr 12	Yr 13	Yr 14	Yr 15	Yr 16	Yr 17	
Ht	255,84	262,08	267,62	272,55	276,93	280,82	284,28	287,36	
Inc	7,01	6,24	5,54	4,93	4,38	3,89	3,46	3,08	

5.2

Height after 10 years
$= 150 + \frac{18\left(1 - \left(\frac{8}{9}\right)^9\right)}{1 - \frac{8}{9}}$
$= 150 - 105{,}8768146 \ldots$
$= 255{,}88$ cm
OR
$= 150 + \frac{18\left(\left(\frac{8}{9}\right)^9 - 1\right)}{\frac{8}{9} - 1}$
$= 150 + 105{,}8768146 \ldots$
$= 255{,}88$ cm

NOTE:
By writing out 9 terms and adding to 150 and answer correct, full marks
Answer only: 2 / 3

✓ $n = 9$ ✓ substitution into sum formula ✓ answer (3)

5.3

Max height = 150 + sum to infinity
$= 150 + \frac{18}{1 - \frac{8}{9}}$
= 150 cm + 162 cm
= 312 cm
The tree will never reach a height of more than 312 cm.

✓ statement ✓ substitution into the sum to infinity formula ✓ max height (3) **[8]**

NOTE:
If a candidate answers in 5.1 that the growth is $18\left(\frac{8}{9}\right)^{n-1} = 18\left(\frac{8}{9}\right)^{16} = 2{,}73$ cm then 1 / 2
The answer for 5.2 as continued accuracy uses $n = 10$,
Height after 10 years
$= 150 + \frac{18\left(1 - \left(\frac{8}{9}\right)^{10}\right)}{1 - \frac{8}{9}} = 150 + 112{,}11 \ldots = 262{,}11$ cm

This is awarded 3/3 as consistent accuracy

- Consistent Accuracy will apply as a general rule.
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6.3

The equation of the inverse of *h* is
$x = y + 2$
$\therefore y = x - 2$

Answer only: Full marks

✓ interchange *x* and *y* ✓ answer (2)

6.4

$g(x) = -\frac{1}{x+1} + 1 = \frac{-1 + x + 1}{x+1} = \frac{x}{x+1}$

$LHS = \frac{x}{x+1} + \frac{\frac{1}{x}}{\frac{1}{x} + 1}$
$= \frac{x}{x+1} + \frac{1}{x+1}$
$= \frac{x+1}{x+1}$
$= 1$

$RHS = \left(\frac{-x}{1-x}\right)\left(\frac{x-1}{(x-1)+1}\right)$
$= \frac{(1-x)x}{(1-x)x}$
$= 1$

LHS = RHS

NOTE:
If substitute a value of *x* and prove it, then 0 / 3

OR

$LHS = g(x) + g\left(\frac{1}{x}\right)$
$= -\frac{1}{x+1} + 1 - \frac{1}{\frac{1}{x} + 1} + 1$
$= -\frac{1}{x+1} + 2 - \frac{x}{1+x}$
$= -\frac{1+x}{1+x} + 2$
$= -1 + 2$
$= 1$

$RHS = g(-x).g(x-1)$
$= \left(-\frac{1}{-x+1} + 1\right)\left(-\frac{1}{x-1+1} + 1\right)$
$= \left(\frac{-1+1-x}{1-x}\right)\left(\frac{-1+x}{x}\right)$
$= \left(\frac{-x}{1-x}\right)\left(\frac{x-1}{x}\right)$
$= \left(\frac{x}{x-1}\right)\left(\frac{x-1}{x}\right)$
$= 1$

LHS = RHS

✓ simplification of $g(x)$ ✓ simplification of LHS ✓ simplification of RHS (3)

✓ 2 substitutions correct.
NOTE: not just rewriting $g(x)$ again ✓ simplification of LHS ✓ simplification of RHS (3)

[13]

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- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 7

7.1	$y \in [-3;3]$ **OR** $-3 \le y \le 3$ **OR** y can be any value from – 3 to 3 **NOTE:** Notation incorrect : 0 / 1	✓ answer (1)
7.2	x–value is 7,37° to the left of 90° B(82,63° ; 0,38) **NOTE:** Answer only : 3 / 3 x-value correct and y-value incorrect : 2 / 3 x-value incorrect and y-value correct : 1 / 3 If decimal part incorrect of x and y-value correct: 2 / 3	✓ method ✓ x-value ✓ y-value (3)
7.3	$\text{Period} = \frac{360°}{3}$ $= 120°$ **NOTE:** Answer only : 2 / 2	✓ $\frac{360°}{3}$ ✓ answer (2)
7.4	$x = -180°$	✓ ✓answer (2) **[8]**

QUESTION 8

8.1	$x > 0$ **OR** $x \in (0;\infty)$	✓ answer (1)
8.2	$y = 2^{-x}$ **OR** $y = \left(\frac{1}{2}\right)^x$	✓ answer (1)
8.3	$y = 0$	✓ answer (1)
8.4.1	Reflect the graph of f over the x-axis OR For each point the y-coordinate changes sign. **NOTE:** Reflect only : 0 / 1	✓ answer (1)
8.4.2	Reflect the graph of f over the line $y = x$. Then shift the graph down 5 units	✓✓ answer ✓ answer (3)

- Consistent Accuracy will apply as a general rule.
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- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

8.4.2 contd	**OR** Sketch the graph of the inverse of f. Shift the graph of the inverse of f down by 5 units. **OR** Shift the graph 5 units LEFT. Reflect the graph over the line $y = x$.	
8.5	$\log_2 x < 3$ $-\log_2 x > -3$ For $-\log_2 x = -3$ $2^3 = x$ $x = 8$ $f(x) > -3$ $0 < x < 8$ or $x \in (0;8)$ **NOTE:** Notation incorrect: Answer $x < 8$: 2 / 3 Answer only correct: 3 / 3	✓ multiplication by – 1 ✓ Notation ✓ critical values (3) **[10]**

QUESTION 9

Penalise ONCE in question 9 for early rounding off.

9.1	$A = P(1-i)^n$ $15000 = 24000(1-0,18)^n$ $0,625 = (0,82)^n$ $n = \frac{\log 0,625}{\log 0,82}$ $= 2,37$ years **NOTE:** If subs A and P incorrectly: Answer would be $n = -2,37$ years $\therefore n = 2,37$ years: 2 / 4 If subs A and P incorrectly: Answer would be $n = -2,37$ years : 1 / 4 Answer $n = 2,4$ years 4 / 4 Answer rounded to 3 years and all calculations shown and $n = 2,37$ shown: 4/4 Answer rounded to 3 years and $n = 2,37$ not shown: 3 / 4	✓ substitution ✓ simplification ✓application of logs ✓ answer (4) Incorrect formula: 0/4
9.2.1	$130\,000\left(1+\frac{0,18}{12}\right)^2$ $= 130000(1,015)^2$ $= \text{R } 133\,929,25$ **NOTE:** – 1 per error for incorrect substitution to a max of 2 marks	✓✓ substitution ✓ answer (3) Incorrect formula: 0/3

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

9.2.2(a)	$133\,929,25 = \frac{x[1-(1,015)^{-54}]}{0,015}$ $2008,93875 = x[1-(1,015)^{-54}]$ $x = \text{R } 3636,36$ **OR** $133\,929,25\left(1+\frac{0,18}{12}\right)^{54} = \frac{x\left[\left(1+\frac{0,18}{12}\right)^{54}-1\right]}{\frac{0,18}{12}}$ $299255,2087 = 82,29517136...x$ $x = \text{R } 3636,36$ **OR** $130000\left(1+\frac{0,18}{12}\right)^{56} = \frac{x\left[\left(1+\frac{0,18}{12}\right)^{54}-1\right]}{\frac{0,18}{12}}$ $299255,2087 = 82,29517136...x$ $x = \text{R } 3636,36$	✓ $n = 54$ ✓ substitution of 133 929,25 ✓ answer (3) ✓ $n = 54$ ✓ substitution of 133 929,25 ✓ answer (3) ✓ $n = 54$ ✓ $130000\left(1+\frac{0,18}{12}\right)^{56}$ ✓ answer (3)
9.2.2(b)	Total = 3636,36 × 54 = R196 363,66 **NOTE:** Accept answer = R 196	✓answer (1)
9.2.3	$130\,000 = \frac{x[1-(1,015)^{-54}]}{0,015}$ $1950 = x[1-(1,015)^{-54}]$ $x = \text{R } 3529,68$ Total payments = R 3529,68 × 54 = R 190 602,72 **OR**	✓ 130 000 ✓ $i = 0,015$ ✓ answer 3529,68 ✓ answer R 190 602.72 (4)

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it. mark the deleted attempt.

9.2.3 contd	$130\,000\left(1+\frac{0,18}{12}\right)^{54} = \frac{x\left[\left(1+\frac{0,18}{12}\right)^{54}-1\right]}{\frac{0,18}{12}}$ $290475,5842 = 82,29517136...x$ $x = \text{R } 3529,68$ Total payments = R 3529,68 × 54 = R 190 602,72 **OR** $130\,000\left(1+\frac{0,18}{12}\right)^{55} = \frac{x\left(1+\frac{0,18}{12}\right)\left[\left(1+\frac{0,18}{12}\right)^{54}-1\right]}{\frac{0,18}{12}}$ $290475,5842 = 82,29517136...x$ $x = \text{R } 3529,68$ Total payments = R 3529,68 × 54 = R 190 602,72 **NOTE:** Disregard the cents values if they are incorrect.	✓ $130\,000\left(1+\frac{0,18}{12}\right)^{54}$ ✓ $i = 0,015$ ✓ answer 3529,68 ✓ answer R 190 602,72 (4) ✓ $130\,000\left(1+\frac{0,18}{12}\right)^{55}$ ✓ $i = 0,015$ ✓ answer 3529,68 ✓ answer R 190 602,72 (4)
9.2.4	R196 363,66 – R190 602,72 =R5 760,96	✓ answer (1) **[16]**

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 10

10.1

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{-2(x+h)^2+3-(-2x^2+3)}{h}$$
$$= \lim_{h\to 0}\frac{-2x^2-4xh-2h^2+3+2x^2-3}{h}$$
$$= \lim_{h\to 0}\frac{h(-4x-2h)}{h}$$
$$= \lim_{h\to 0}(-4x-2h)$$
$$= -4x$$

✓ $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$
✓ $-2(x+h)^2+3$
✓ simplification
✓ simplification
✓ answer (5)

NOTE:
Penalty 1 mark only for incorrect notation (lim missing or = in incorrect place)

Answer only : 0 / 5

Cannot give mark for answer if the answer is incorrect according to the working out, even if the answer is given as – 4*x*.

10.2

$$y = x^2 - \frac{1}{2x^3}$$
$$y = x^2 - \frac{1}{2}x^{-3}$$
$$\frac{dy}{dx} = 2x + \frac{3}{2}x^{-4}$$

OR

$$\frac{dy}{dx} = 2x + \frac{3}{2x^4}$$

OR

$$\frac{dy}{dx} = 2x - (-3)\frac{1}{2}x^{-4}$$

✓ $2x$
✓ $+\frac{3}{2}x^{-4}$ (2) **[7]**

- Consistent Accuracy will apply as a general rule.
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- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 11

11.1

$$0 = -x^3 + x^2 + 8x - 12$$
$$x^3 - x^2 - 8x + 12 = 0$$
$$(x-2)(x^2+x-6) = 0$$
$$(x-2)(x-2)(x+3) = 0$$
$$x = 2 \text{ or } x = -3$$

x-intercepts are (2 ; 0) and (– 3 ; 0)

OR

$$0 = -x^3 + x^2 + 8x - 12$$
$$x^3 - x^2 - 8x + 12 = 0$$
$$(x+3)(x^2-4x+4) = 0$$
$$(x+3)(x-2)(x-2) = 0$$
$$x = 2 \text{ or } x = -3$$

x-intercepts are (2 ; 0) and (– 3 ; 0)

✓ any one of factors
✓ quadratic factor
✓ linear factors
✓✓ x-answers (5)

11.2

$$f'(x) = -3x^2 + 2x + 8$$
$$0 = 3x^2 - 2x - 8$$
$$0 = (x-2)(3x+4)$$
$$x = 2 \text{ or } x = -\frac{4}{3}$$

turning points are (2 ; 0) and $\left(-\frac{4}{3};-\frac{500}{27}\right)$

OR (2 ; 0) and (– 1,33 ; – 18,52)

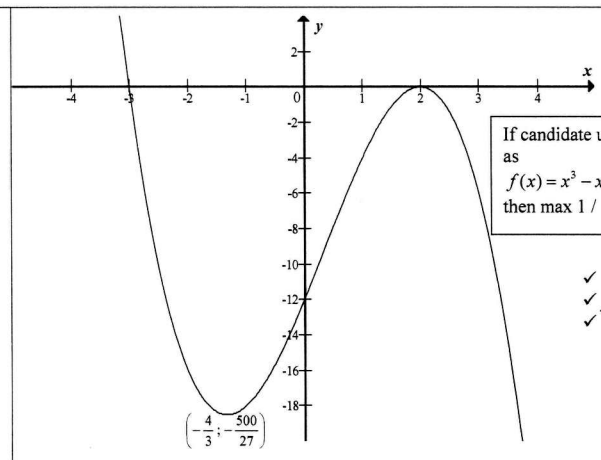
✓ $f'(x) = 0$
✓ $-3x^2+2x+8=0$ or $3x^2-2x-8=0$
✓ factors
✓ x-values
✓ y-values (5)

NOTE:
If = 0 is omitted in 11.2: penalty 1 mark

If not in coordinate form but coordinates implied: OK

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

11.3



If candidate used function as $f(x) = x^3 - x^2 - 8x + 12$ then max 1 / 3

✓ shape
✓ y-intercept
✓ turning pts (3)

11.4

$f''(x) = 0$ or $f''(x) = 0$
$6x - 2 = 0$ $-6x + 2 = 0$
$x = \frac{1}{3}$ $x = \frac{1}{3}$

OR

$$x = \frac{2-\frac{4}{3}}{2}$$
$$x = \frac{1}{3}$$

Note:
If write down $f''(x) = 6x - 2$ or $f''(x) = -6x + 2$ then 1 / 2

✓ method
✓ answer
Answer only: Full marks (2)

11.5

(2 ; –3) and $\left(-\frac{4}{3};-\frac{581}{27}\right)$

OR

(2 ; –3) and (–1,33 ; – 21,52)

✓✓ each answer (2) **[17]**

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 12

12.1

$s(0) = 5(0)^3 - 65(0)^2 + 200(0) + 100$
$= 100$ metres

NOTE:
If subs $t = 8$, then answer = 100: 0 / 2

✓ $t = 0$
✓ answer (2)
Answer only: full marks

12.2

$s(t) = 5t^3 - 65t^2 + 200t + 100$
$s'(t) = 15t^2 - 130t + 200$
$s'(4) = 15(4)^2 - 130(4) + 200$
$= -80$ metres per minute

NOTE:
If used average rate of change between $t = 0$ and $t = 4$: 0 / 3
If subs $t = 4$ into $s(t)$: 0 / 3

✓ $s'(t) = 15t^2 - 130t + 200$
✓ substitution $t = 4$
✓ answer (– 80) (3)

12.3

The height of the car above sea level is decreasing at 80 metres per minute and the car is travelling downwards hence it is a negative rate of change.

OR

The **vertical** velocity of the car at $t = 4$ is 80 metres per minute.

NOTE:
Mark this CA even if answer to QUESTION 12.2 is completely inaccurate.

✓ speed 80 metres per minute
✓ downwards (2)

12.4

$s'(t) = 15t^2 - 130t + 200$
$s''(t) = 30t - 130$
$130 = 30t$
$t = 4{,}3\dot{3}$ minutes

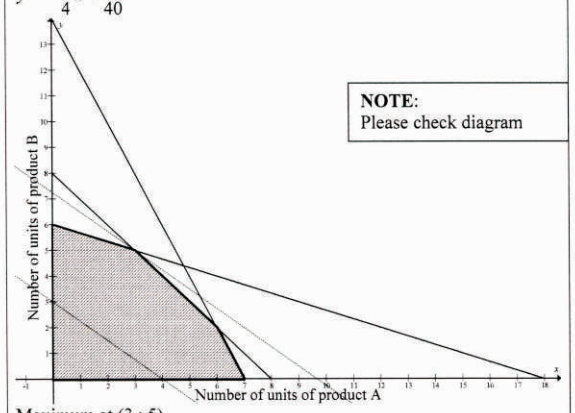
OR

$$t = \frac{-(-130)}{2(15)}$$
$t = 4{,}3\dot{3}$ minutes

✓ $s''(t) = 30t - 130$
✓ $s''(t) = 0$
✓ answer (3) **[10]**

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 13

13.1	$x+3y \le 18$ $x+y \le 8$ $2x+y \le 14$ $x, y \ge 0$ **OR** $6x+18y \le 108$ $8x+8y \le 64$ $14x+7y \le 98$ $x, y \ge 0$ **OR** $y \le -\frac{1}{3}x+6$ $y \le -x+8$ $y \le -2x+14$ $x, y \ge 0$	**NOTE:** If written as equations (inequality omitted): max 6 / 7 If inequalities sign the wrong way round: max 6 / 7 One should note that x and y should be counting numbers	✓✓ answer ✓✓ answer ✓✓ answer ✓ answer (7)
13.2	$P = 30x + 40y$	**NOTE:** If P = 30A + 40B then 1 / 2	✓✓ answer (2)
13.3	$y = -\frac{3}{4}x + \frac{P}{40}$  Maximum at (3 ; 5)	**NOTE:** Please check diagram	✓✓ answer (2) Answer only : Full marks
13.4	$-2 < m < -1$	**NOTE:** accept $1 < m < 2$: 2 / 2. If ≤ signs used then max 1 / 2	✓✓answer (2) **[13]**

Notes

Notes

Notes

education
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EDUCATION FOR ALL

education
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Grade 12 Education Supplement 2010

Mathematic Paper 02
November 2009

Mathematics

Mathematic Paper 02
November 2009

Together Educating the Nation

QUESTION 1

The data below shows the total monthly rainfall (in millimetres) at Cape Town International Airport for the year 2002.

Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
60,9	14,9	9,3	28,0	71,9	76,4	98,2	65,7	26,1	32,5	23,6	15,0

[Source: www.1stweather.com]

1.1 Determine the mean monthly rainfall for 2002. (2)

1.2 Write down the five-number summary for the data. (5)

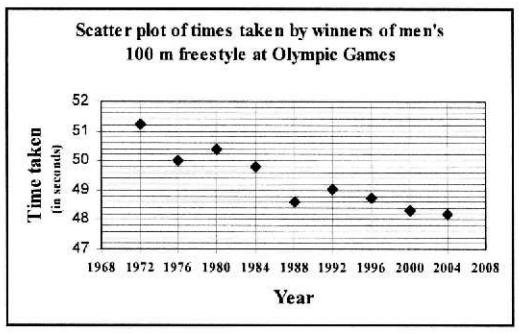
1.3 Draw a box and whisker diagram for the data on DIAGRAM SHEET 1. (3)

1.4 By making reference to the box and whisker diagram, comment on the spread of the rainfall for the year. (2)

1.5 Calculate the standard deviation for the data. (3)
[15]

QUESTION 2

The scatter plot below represents the times taken by the winners of the men's 100 m freestyle swimming event at the Olympic Games from 1972 to 2004. The data was obtained from www.databaseOlympics.com.



2.1 Indicate whether a linear, quadratic or exponential function best fits the data. (1)

2.2 Draw a line of best fit for the data on the graph provided on DIAGRAM SHEET 1. (2)

2.3 Describe the trend that is observed in these times. (1)

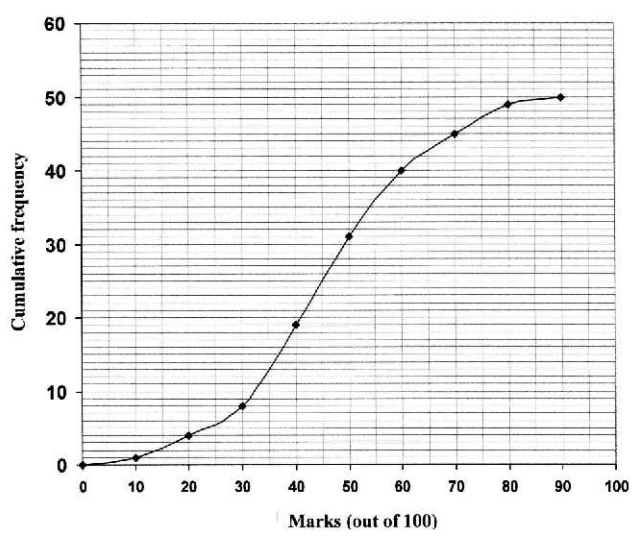
2.4 Give ONE reason for this trend. (1)

2.5 What can be said about the efforts of the winners in the years 1976 and 1988? (2)

2.6 Use your line of best fit to predict the winning time for 2008. (1)
[8]

QUESTION 3

The ogive (cumulative frequency graph) shows the performance of students who took a test in basic programming skills. The test had a maximum of 100 marks.



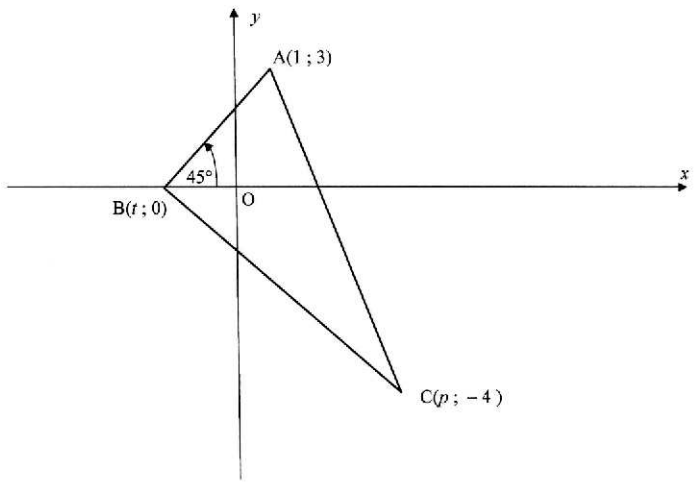
3.1 How many students took the test? (1)

3.2 Only the top 25% of the students are allowed to do an advanced course in programming. Determine the cut-off mark to determine the top 25%. (1)

3.3 Construct a frequency table for the information given in the ogive on DIAGRAM SHEET 2. Complete the table with the information. (3)
[5]

QUESTION 4

ABC is a triangle with vertices A(1 ; 3), B(t ; 0) and C(p ; −4), with $p > 0$, in a Cartesian plane. AB makes an angle of 45° with the positive x-axis. AC = $\sqrt{50}$.



4.1 Determine the gradient of AB. (2)

4.2 Calculate the value of t. (2)

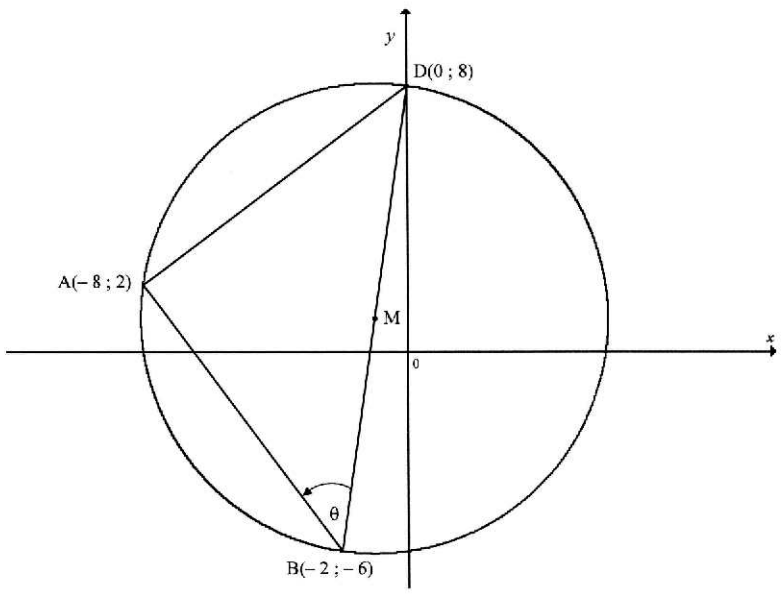
4.3 Calculate p, the x-coordinate of point C. (4)

4.4 Hence, determine the midpoint of BC. (2)

4.5 Determine the equation of the line parallel to AB, passing through point C. (3)
[13]

QUESTION 5

A(−8 ; 2), B(−2 ; −6) and D(0 ; 8) are the vertices of a triangle that lies on the circumference of a circle with diameter BD and centre M, as shown in the figure below.



5.1 Calculate the coordinates of M. (2)

5.2 Show that (−8 ; 2) lies on the line $y = 7x + 58$. (1)

5.3 What is the relationship between the line $y = 7x + 58$ and the circle centred at M? Motivate your answer. (5)

5.4 Calculate the lengths of AD and AB. (4)

5.5 Prove $D\hat{A}B = 90°$. (3)

5.6 Write down the size of angle θ. (1)

5.7 A circle, centred at a point Z inside ΔABD, is drawn to touch sides AB, BD and DA at N, M and T respectively. Given that BMZN is a kite, calculate the radius of this circle. A diagram is provided on DIAGRAM SHEET 2. (6)
[22]

QUESTION 6

6.1 ABC is a triangle that has an area of 5 square units. $\Delta A'B'C'$ is an enlargement of ΔABC through the origin by a scale of 2.

6.1.1 Determine the area of $\Delta A'B'C'$. (2)

6.1.2 Write down the general rule for the transformation from ΔABC to $\Delta A'B'C'$. (2)

6.1.3 The vertices of ΔABC are A(–1 ; 4), B(–1 ; 2) and C(4 ; 4). Use the grid provided on DIAGRAM SHEET 3 to draw $\Delta A'B'C'$. (3)

6.1.4 Comment on the rigidity of the transformation from ΔABC to $\Delta A'B'C'$. (2)

6.2 A quadrilateral EFGH is transformed to its image $E'''F'''G'''H'''$ in the following way:

- First, reflect EFGH about the line $y = x$.
- Then, rotate this image through 90° in a clockwise direction about the origin.
- The second image has a translation of 2 units to the left and 3 units down to obtain $E'''F'''G'''H'''$.

Write down the general rule of the transformation of EFGH to $E'''F'''G'''H'''$. (6)
[15]

QUESTION 7

$A'(-1-\sqrt{2}\,;\,1-\sqrt{2})$ is the image of point A(p ; q), after point A has been rotated about the origin in an anti-clockwise direction, through an angle of 135°.

7.1 T(x ; y) is rotated about the origin through an angle of θ in an anti-clockwise direction. Write down a formula to determine the coordinates of T', the image of T. (2)

7.2 Write down the coordinates of A' in terms of p, q and 135°. (2)

7.3 Hence, or otherwise, calculate p and q. (Leave your answer in surd form.) (7)
[11]

QUESTION 8

Given: $\sin\alpha = \frac{8}{17}$ where $90° \le \alpha \le 270°$

With the aid of a sketch and without the use of a calculator, calculate:

8.1 $\tan\alpha$ (3)

8.2 $\sin(90° + \alpha)$ (2)

8.3 $\cos 2\alpha$ (3)
[8]

QUESTION 9

9.1 Simplify completely:

$\sin(90° - x)\cos(180° - x) + \tan x.\cos(-x)\sin(180° + x)$ (7)

9.2 Prove, without the use of a calculator, that $\frac{\sin 190°\cos 225°\tan 390°}{\cos 100°\sin 135°} = -\frac{1}{\sqrt{3}}$ (7)

9.3 Determine the general solution of $\sin x + 2\cos^2 x = 1$. (7)
[21]

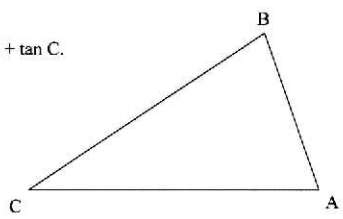
QUESTION 10

10.1 Using the expansions for sin(A + B) and cos(A + B), prove the identity of:

$\frac{\sin(A+B)}{\cos(A+B)} = \frac{\tan A + \tan B}{1 - \tan A.\tan B}$ (3)

10.2 If $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$, prove in any ΔABC that

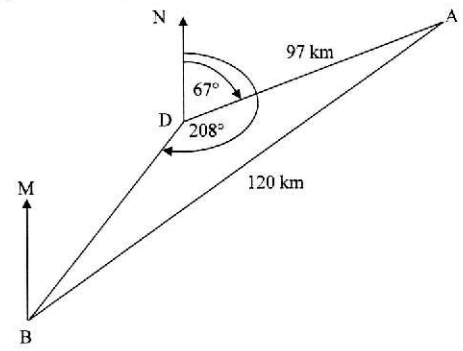
tan A.tan B.tan C = tan A + tan B + tan C.



(4)
[7]

QUESTION 11

11.1 Two ships, A and B, are 120 km apart. Ship A is at a bearing of 67° from D and 97 km away from D. DN points due north. Ship B is at a bearing of 208° from D. A diagram is provided on DIAGRAM SHEET 3.

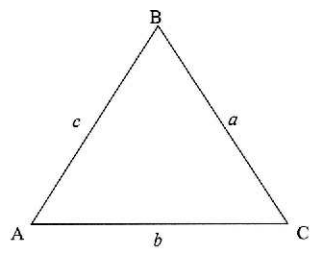


11.1.1 Determine the bearing of Ship A from Ship B, that is $M\hat{B}A$, when BM || DN. (6)

11.1.2 If Ship B travels due north, and Ship A travels due south, then at some instant of time Ship A is due east of Ship B.

Calculate the distance between the two ships at that instant. (3)

11.2 Triangle ABC is isosceles with AB = BC.



Prove that $\cos B = 1 - \frac{b^2}{2a^2}$ (4)
[13]

QUESTION 12

Given: $g(x) = 2\cos(x - 30°)$

12.1 Sketch the graph of g for $x \in [-90° ; 270°]$ on DIAGRAM SHEET 4. (2)

12.2 Use the symbols A and B to plot the two points on the graph of g for which $\cos(x - 30°) = 0{,}5$ (2)

12.3 Calculate the x-coordinates of the points A and B. (3)

12.4 Write down the values of x, where $x \in [-90° ; 270°]$ and $g'(x) = 0$. (2)

12.5 Use the graph to solve for x, $x \in [-90° ; 270°]$, where $g(x) < 0$ (3)
[12]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\sum_{i=1}^{n} (a+(i-1)d) = \frac{n}{2}(2a+(n-1)d)$

$\sum_{i=1}^{n} ar^{i-1} = \frac{a(r^n-1)}{r-1}$; $r \neq 1$ $\sum_{i=0}^{\infty} ar^{i-1} = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC:

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha+\beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha-\beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha-\beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum (x-\bar{x})(y-\bar{y})}{\sum (x-\bar{x})^2}$

CENTRE NUMBER:

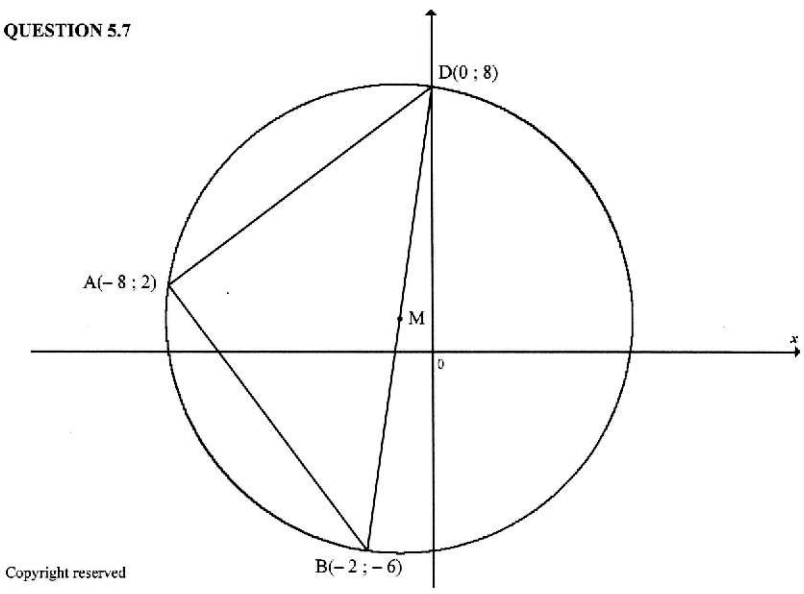
EXAMINATION NUMBER:

DIAGRAM SHEET 2

QUESTION 3.3

Marks (out of 100)	Frequency (f)

QUESTION 5.7

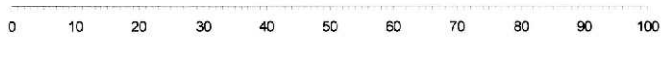


CENTRE NUMBER:

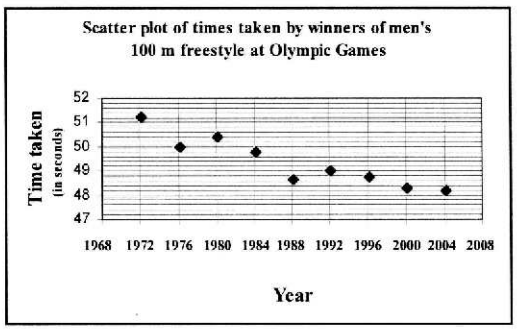
EXAMINATION NUMBER:

DIAGRAM SHEET 1

QUESTION 1.3



QUESTION 2.2

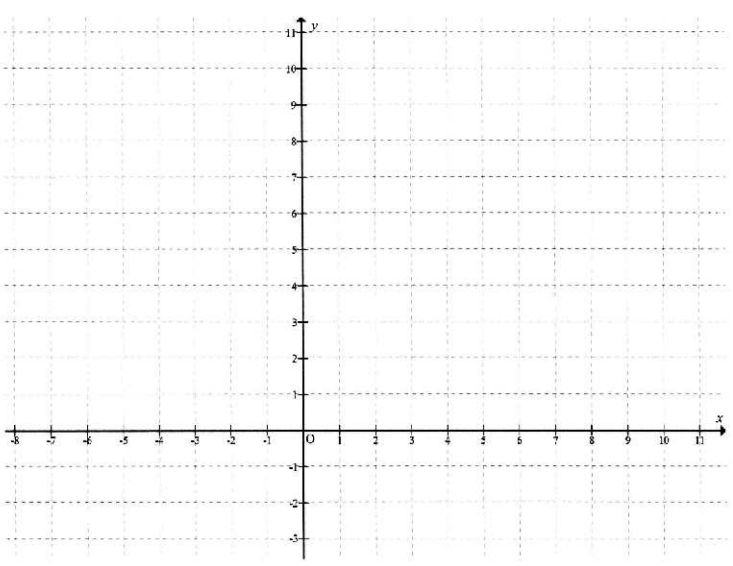


SENTRUMNOMMER:

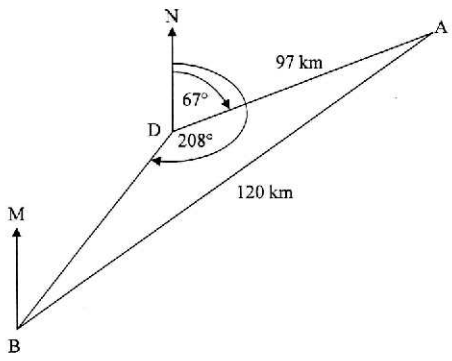
EKSAMENNOMMER:

DIAGRAM SHEET 3

QUESTION 6.1.3



QUESTION 11



Mathematics/P2 DoE/November 2009(1)

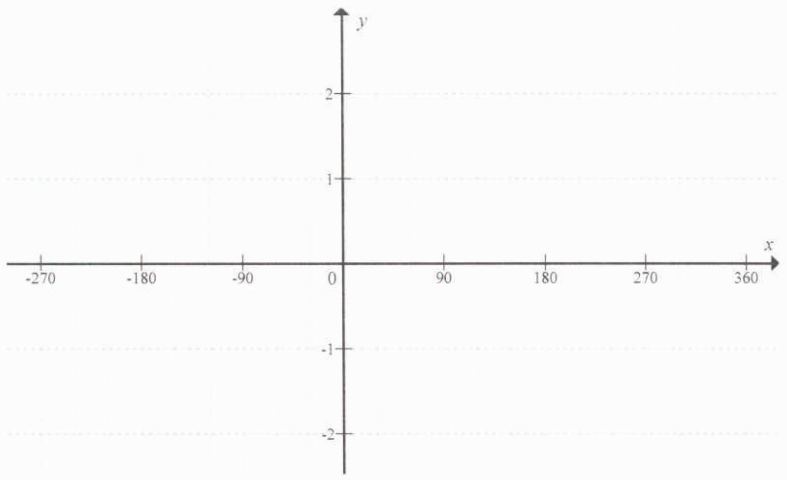
NSC

SENTRUMNOMMER:

EKSAMENNOMMER:

DIAGRAM SHEET 4

QUESTION 12.1



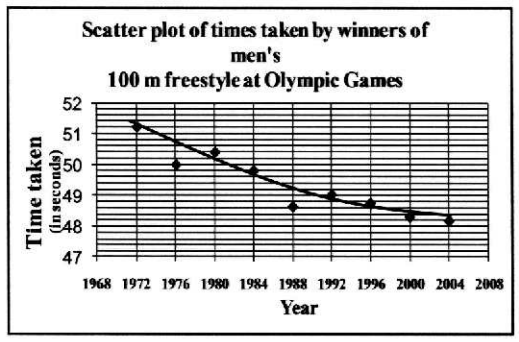
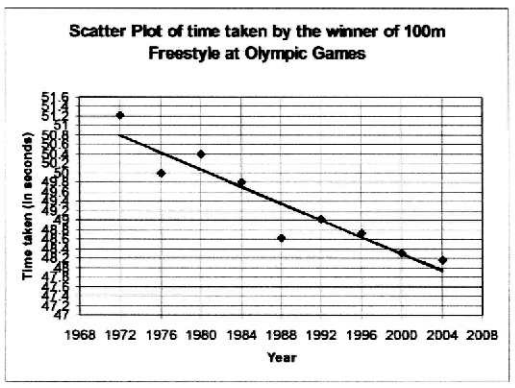
- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 1

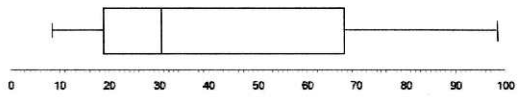
1.1	Mean = $\frac{522,5}{12} = 43,5$ **ANSWER ONLY:** Full marks	✓ 522,5 ✓ answer (2) No penalty for Rounding: Accept 43,54 ; 44
1.2	Ordered Data 9,3 14,9 15 23,6 26,1 28 32,5 60,9 65,7 71,9 76,4 98,2 Median = $\frac{28+32,5}{2} = 30,3$ Lower quartile = $\frac{15+23,6}{2} = 19,3$ Upper quartile = $\frac{65,7+71,9}{2} = 68,8$ The five number summary is (9,3 ; 19,3 ; 30,25 ; 68,8 ; 98,2) OR If they use the formula: Ordered Data 9,3 14,9 15 23,6 26,1 28 32,5 60,9 65,7 71,9 76,4 98,2 Position of median: $P_{50} = \frac{12+1}{2} = 6,5$ $\therefore Q_2 = \frac{28+32,5}{2} = 30,3$ Position of lower quartile: $P_{25} = \frac{13}{4}$ $\therefore Q_1 = 15 + (0,25(23,6-15)) = 17.15$ Position of upper quartile: $P_{75} = 0,75(13) = 9,75$ $\therefore Q_3 = 65,7 + (0,75(71,9-65,7)) = 70,35$ Min = 9,3 Max = 98,2 Accept any one of these five number summaries: (9,3 ; 19,3 ; 30,3 ; 68,8 ; 98,2) (9,3 ; 15 ; 30,3 ; 71,9 ; 98,2) (9,3 ; 17,2 ; 30,3 ; 70,4 ; 98,2)	✓ 9,3 ✓ 19,3 ✓ 30,3 ✓ 68,8 ✓ 98,2 (5) If indicated on the box and whisker diagram in 1.3 – 5 marks

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 2

2.1	Linear or Exponential	✓ answer (1)
2.2	  For this set of data we will accept the straight line.	✓ ✓line of best fit (2)

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

1.3	 Note: If just a box and whisker without any reference to the numbers: 1/3	✓ minimum and maximum values ✓ quartiles and median ✓ whiskers with median line (3)
1.4	The data is skewed to the right (positively skewed). This suggests that there was a large difference between the median and the maximum rainfall (some months had exceptionally high rainfall in that year). *Die data is skeef na regs (positief skeef)* *Dit dui daarop dat daar 'n groot verskil is tussen die mediaan en die maksimum reënval (sommige maande het ongewoon hoë reënval gehad gedurende die jaar.*	✓ ✓ comment about rainfall. (2) Note: Skewed to right 1/2 ✓ ✓ verwysing na reënval (2)
1.5	By using the calculator, $\sigma = 28,19$. (28,19058256) **OR Pen and Paper method (not recommended)** Mean = 43,54 (43,54166667) $\sigma = \sqrt{\frac{9536,509}{12}} = 28,19$ (28,19059.....)	✓✓✓answer Accept: 28 ; 28,2 ; 28,1 (3) ✓headings correct ✓sum of the squares of the mean deviations ✓ answer (3) **[15]**

x	$x-\bar{x}$	$(x-\bar{x})^2$
60,9	17,36	301,3696
14,9	-28,64	820,2496
9,3	-34,24	1172,378
28,0	-15,54	241,4916
71,9	28,36	804,2896
76,4	32,86	1079,78
98,2	54,66	2987,716
65,7	22,16	491,0656
26,1	-17,44	304,1536
32,5	-11,04	121,8816
23,6	-19,94	397,6036
15,0	-28,54	814,5316
Sum		9536,509

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
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2.3	The scatter plot shows an overall decrease in the time taken by the winner since 1972. *Die spreidiagram dui 'n algehele afname in tye aangeteken deur die wenners vanaf 1972.* OR Times are faster. *Tye is vinniger.* OR Negative correlation between year and time. *Negatiewe korrelasie tussen jaar en tyd.*	✓ decrease/afname (1)
2.4	The top athletes of the world have turned professional. This allows them to train at the best facilities and receive the best coaching available. Also, equipment manufacturers are in competition with each other. In this case, manufacturers are designing swimsuits that assist swimmers Swimmers train harder and put in more effort. *Die top atlete van die wêreld het professionele atlete geword. Dit laat hulle toe om by die beste fasiliteite te oefen en die beste afrigting te ontvang.* *Vervaardigers van voorraad is in kompetisie met mekaar. Hul onwerp dus swembroeke wat die swemmers help.* *Swemmers oefen harder en gebruik meer tyd om te oefen.*	✓ any acceptable reason relating to the trend (1) ✓ *enige aanvaarbare rede wat verband hou met die neiging.* (1)
2.5	In the context of the times around these two observations, one can consider the efforts of 1976 and 1988 to be outliers. This shows that these athletes were exceptionally good swimmers at the time. *Binne die konteks van tye gedurende hierdie twee waarnemings, kan die poging van 1976 and 1988 gesien word as uitskieters. Dit dui daarop dat hierdie atlete uitstekende swemmers was daardie tyd.*	✓✓acceptable reason in context (2) ✓✓aanvaarbare rede binne die konteks (2)
2.6	Winning time of 2008 is expected to be about 47,6 seconds. Accept answer from candidate's graph.	✓answer from graph (1) **[8]**

QUESTION 3

3.1	50	✓ answer (1)
3.2	Cut–off mark of 56% (37 students)or 58% (38 students) Accept interval: 55% - 60%	✓ answer read off from ogive (1)

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3.3

Marks (out of 100)	Frequency (f)
$0 \le$ marks <10	1
$10 \le$ marks <20	3
$20 \le$ marks <30	4
$30 \le$ marks <40	11
$40 \le$ marks <50	12
$50 \le$ marks <60	9
$60 \le$ marks <70	5
$70 \le$ marks <80	4
$80 \le$ marks <90	1
$90 \le$ marks <100	0

✓ class intervals
Accept
0 – 10 ; 10 – 20
Or
0 < marks ≤ 10
Or
Between 0 and 10
Or
From 0 to 10

If the intervals not in tens, the mark for intervals not given

✓method
✓ accuracy of five answers
(3)
[5]

QUESTION 4

4.1	$\tan 45° = m_{AB}$ $= 1$ OR $m_{AB} = \frac{3-0}{1-t} = \frac{3}{1-t}$	✓ $\tan 45°$ ✓ answer (2) Answer only: full marks
4.2	$\frac{3-0}{1-t} = \tan 45° = 1$ $1-t = 3$ $t = -2$ OR $y = mx + c$ $3 = (1)(1) + c$ $c = 2$ $y = x + 2$ $(t;0)$ in $y = mx + 2$ $0 = t + 2$ $t = -2$	✓equating ✓value (2) ✓c=2 ✓value (2) Answer only: full marks

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4.3	$\sqrt{(1-p)^2 + (3+4)^2} = \sqrt{50}$ $(1-p)^2 + (3+4)^2 = 50$ $1 - 2p + p^2 + 49 = 50$ $p^2 - 2p = 0$ $p(p-2) = 0$ $p \ne 0$ or $p = 2$ OR $(1-p)^2 + (3+4)^2 = 50$ $(1-p)^2 = 50 - 49$ $(1-p)^2 = 1$ $1 - p = 1$ or $1 - p = -1$ $p \ne 0$ $p = 2$ OR *Let* $p = 2$ $AC = \sqrt{(1-2)^2 + (3+4)^2}$ $= \sqrt{1+49}$ $= \sqrt{50}$ which is true $\therefore p = 2$	✓ substitution into distance formula ✓ expansion ✓factors ✓answer Note: If an answer was not chosen: 3/4 (4) ✓ substitution into distance formula ✓ expansion ✓factors ✓answer (4) If gradient of BC assumed as -1 and p calculated correctly: 0/4 Answer only: 1/4 ✓ substitution into distance formula ✓ $\sqrt{50}$ ✓which is true(justification) ✓answer (4) If equating to $\sqrt{50}$ from the start, then 3/4
4.4	midpoint of BC $= \left(\frac{-2+2}{2}; \frac{0-4}{2}\right)$ midpoint of BC $= (0; -2)$	✓ x-value ($x = \frac{t+p}{2}$) ✓y-value (2)
4.5	Gradient of line = $m_{AB} = 1$ Equation of line is: $y + 4 = 1(x - 2)$ $y = x - 6$ OR $y = mx + c$ $y = x - p - 4$	✓ gradients are equal ✓substitution of $(p;-4)$ ✓equation in any form (3) **[13]**

- Consistent Accuracy will apply as a general rule.
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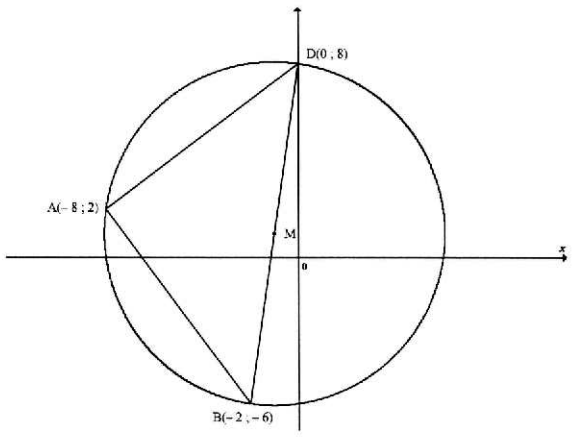
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 5



5.1	Midpoint BD $\left(\frac{0-2}{2}; \frac{8-6}{2}\right)$ $= (-1; 1)$	✓x-coordinate ✓y-coordinate (2)
5.2	$y = 7(-8) + 58$ $= 2$ $\therefore$ A lies on the line.	✓substitution (1) Substitute both at the same time with justification (1)
5.3	The line $y = 7x + 58$ is a tangent to the circle at A. $m_{line} = 7$ $m_{AM} = \frac{2-1}{-8-(-1)} = -\frac{1}{7}$ $m_{line} \times m_{AM} = 7 \times -\frac{1}{7} = -1$ $\therefore$ AM $\perp$ to the line OR	✓relationship ✓✓ $m_{AM} = \frac{2-1}{-8-(-1)} = -\frac{1}{7}$ ✓ $m_{line} = 7$ ✓product (5)

NOTE:
$m_{line} = 7$ and CA gradient of AM then no relationship: 4/5

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- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

5.3 contd	**OR** $m_{BD} = 7$ $m_{line} = 7$ $\therefore$ line // diameter **OR** $(x+1)^2 + (y-1)^2 = 50$ $x^2 + 2x + 1 + y^2 - 2y + 1 = 50$ $x^2 + 2x + 1 + (7x + 58)^2 - 2(7x + 58) + 1 = 50$ $x^2 + 2x + 1 + 49x^2 + 812x + 3364 - 14x - 116 + 1 = 50$ $50x^2 + 800x + 3200 = 0$ $x^2 + 16x + 64 = 0$ $(x+8)^2 = 0$ $x = -8$ $y = 2$ $y = 7x + 58$ is a tangent to the circle	✓✓ $m_{BD} = 7$ ✓ $m_{line} = 7$ ✓✓conclusion (5) Note: Only lines parallel 4/5 ✓ circle equation ✓substitution of $y = 7x + 58$ ✓standard form ✓ answer ✓ tangent (5)
5.4	AD $= \sqrt{(8-2)^2 + (0+8)^2}$ $= \sqrt{36+64}$ $= 10$ AB $= \sqrt{(2+6)^2 + (-8+2)^2}$ $= \sqrt{64+36}$ $= 10$	✓ substitution ✓ answer ✓ substitution ✓ answer (4) Note: Answers $\sqrt{10}$ then 3/4

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5.5	$m_{AD} = \frac{8-(2)}{0-(-8)}$ $m_{AD} = \frac{3}{4}$ $m_{AB} = \frac{2-(-6)}{-8-(-2)}$ $= -\frac{4}{3}$ $m_{AB}.m_{AD} = -\frac{4}{3} \times \frac{3}{4}$ $= -1$ $D\hat{A}B = 90°$ **OR** $BD^2 = (8+6)^2 + (0+2)^2$ $= 200$ $= AD^2 + AB^2$ $\therefore D\hat{A}B = 90°$ **OR** $a^2 = b^2 + d^2 - 2(b)(d)\cos A$ $200 = 100 + 100 - 2(10)(10)\cos A$ $0 = -200\cos A$ $A = 90°$ **OR** $(AD)^2 = 100$ $(AB)^2 = 100$ $BD^2 = (-2-0)^2 + (-6-8)^2$ $= 4 + 196$ $= 200$ $\therefore BD^2 = AD^2 + AB^2$ $\therefore D\hat{A}B = 90°$ (Pyth) **OR** $\hat{A} = 90°$ (angles in semi - circle)	✓ gradient of AD ✓ gradient of AB ✓ PRODUCT (3) ✓ distance formula ✓ Pythagoras ✓ conclusion (3) ✓ cos rule ✓ substitution ✓ conclusion (3) ✓ $BD^2 = 200$ ✓ $BD^2 = AD^2 + AB^2$ ✓ conclusion (3) ✓✓reason (3)
5.6	$\theta = 45°$	✓ answer (1)

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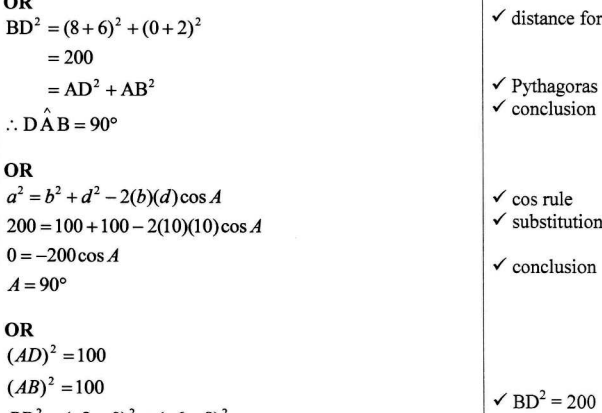

5.7	Let the radius of circle TNM be r NB = BM (properties of a kite) AN = TZ = r (TZNA is a square) NB = $10 - r$ BD = 2MB $\sqrt{(8-(-6))^2 + (0-(-2))^2} = 2(10-r)$ $\sqrt{200} = 2(10-r)$ $10\sqrt{2} = 2(10-r)$ $r = 10 - 5\sqrt{2}$ $= 2,93$ **OR** $Z\hat{M}B = 90°$ $MB = \frac{1}{2}\sqrt{200}$ $= 7,07$ $\frac{ZM}{MB} = \tan 22,5°$ $ZM = 7,07\tan 22,5°$ $= 2,93$ **OR**	✓ NB = BM ✓ AN = TZ = r ✓ NB = $10 - r$ ✓ BD = 2MB ✓BD = $\sqrt{200}$ ✓answer (6) ✓tan radius theorem ✓✓MB ✓✓tan 22,5° ✓answer (6)

- Consistent Accuracy will apply as a general rule.
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- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

5.7 contd	$MB^2 = (-1+2)^2 + (1+6)^2$ $= 1 + 49$ $= 50$ $MB = \sqrt{50}$ $\frac{ZM}{MB} = \tan 22,5°$ $ZM = 7,07\tan 22,5°$ $= 2,93$ **OR** By a well known formula Area $\Delta ABD = r \times$ (semi—perimeter) $\frac{1}{2} \times 10 \times 10 = r \times \frac{1}{2}(20 + \sqrt{200})$ $50 = r(10 + 5\sqrt{2})$ $r = 2,93$ **OR** $MB = \sqrt{50}$ (radius of circle) $NB = \sqrt{50}$ (adjacent sides of kite) AB = 10 AN = $10 - \sqrt{50}$ $= 2,93$ But TANZ is a square ∴ AN = ZN ∴ radius = 2,93	✓✓MB ✓✓tan 22,5° ✓✓answer (6) ✓✓ formula ✓ $\sqrt{200}$ ✓✓ answer (6) ✓MB ✓ NB ✓✓AN = 2,93 ✓ square ✓answer (6)

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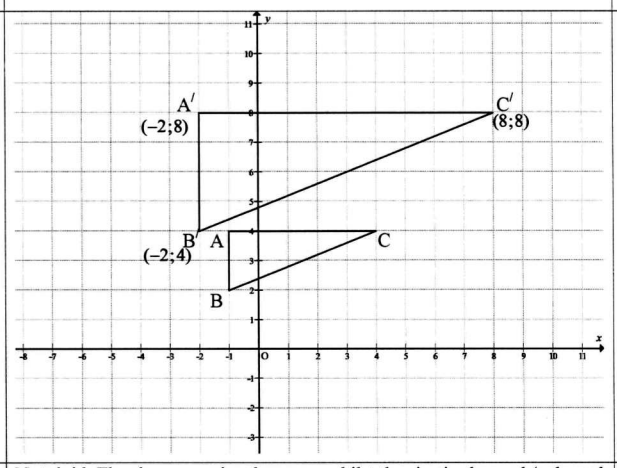
QUESTION 6

6.1.1	4×5 =20 squared units	✓✓answer $2^2 \times 5$ 1/2 If 2×5 = 10 0/2 (2)
6.1.2	$(x\,;y) \rightarrow (2x\,;2y)$ Note: If candidate state: coordinates times two 2/2	✓2x ✓2y (2) If $(kx\,;ky)$:1/2 If $2(x\,;y)$: 2/2
6.1.3		✓ coordinates A' ✓ coordinates B' ✓ coordinates C' (3) If diagram not drawn but coordinates correctly given: 1/3 If coordinates correctly plotted but not joined: 2/3
6.1.4	Not rigid. The shape remains the same, whilst the size is changed /enlarged **Note:** Shape remains the same: 1/2 Only the shape remains the same: 2/2	✓✓same shape and different size (2) not rigid only 2/2 just enlarged 0/2
6.2	Reflection about the line $y = x$: $(x\,;y) \rightarrow (y\,;x)$ Rotate clockwise about the origin: $(y\,;x) \rightarrow (x\,;-y)$ Translate 2 left and 3 down: $(x\,;-y) \rightarrow (x-2\,;-y-3)$ **OR** General rule: $(x\,;y) \rightarrow (x-2\,;-y-3)$	Mark per coordinate ✓✓ reflection ✓✓rotation ✓✓translation (6) **Answer only:** Full marks **[15]**

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OR
The first 2 transformations in the given order is the same as the reflection in the x-axis i.e. $(x\,;y) \rightarrow (x\,;-y)$
Then the translation gives us
$(x\,;y) \rightarrow (x\,;-y) \rightarrow (x-2\,;-y-3)$

NOTE:
If just given: $(x\,;y) \rightarrow (x-2\,;y-3)$: 2/6

If using $(x\,;y) \rightarrow (y\,;x)$ ✓✓
$(x\,;y) \rightarrow (y\,;-x)$ ✓
$(x\,;y) \rightarrow (x-2\,;y-3)$ ✓ throughout :4/6

If learner starts with $(x\,;y)$ and continue to use $(x\,;y)$ for the second and third transformation 4/6

QUESTION 7

7.1	$T'(x\cos\theta - y\sin\theta\,;\, y\cos\theta + x\sin\theta)$	✓ x coordinate ✓ y coordinate (2) Clock-wise formula: 0/2
7.2	$A'(p\cos135° - q\sin135°\,;\, q\cos135° + p\sin135°)$ If clockwise rotation: $A'(p\cos135° + q\sin135°\,;\, q\cos135° - p\sin135°)$	✓ x coordinate ✓ y coordinate (2) CA from 7.1
7.3	$x' = p\cos(135°) - q\sin(135°)$ $-1-\sqrt{2} = -p\cos45° - q\sin45°$ $-1-\sqrt{2} = -p\left(\frac{\sqrt{2}}{2}\right) - q\left(\frac{\sqrt{2}}{2}\right)$ $-1-\sqrt{2} = -\frac{\sqrt{2}}{2}p - \frac{\sqrt{2}}{2}q$.....(1) and $y' = y\cos(135°) + p\sin(135°)$ $1-\sqrt{2} = -q\cos45° + p\sin45°$ $1-\sqrt{2} = q\left(-\frac{\sqrt{2}}{2}\right) + p\left(\frac{\sqrt{2}}{2}\right)$ $1-\sqrt{2} = -\frac{\sqrt{2}}{2}q + \frac{\sqrt{2}}{2}p$.....(2) (1) + (2): $-2\sqrt{2} = -\sqrt{2}q$ $q = 2$	✓ equating ✓ substitution ✓ equating ✓ substitution $\frac{\sqrt{2}}{2}$ ✓ solving simultaneously

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	Substitute $q = 2$ into .....(1) $-1-\sqrt{2} = -\frac{\sqrt{2}}{2}p - \frac{\sqrt{2}}{2}(2)$ $-1 = -\frac{\sqrt{2}}{2}p$ $p = \sqrt{2}$ $\therefore A = (\sqrt{2}\,;2)$ Note: If not left in surd form: 6/7	✓ answer for q ✓ answer for p (7)
	OR $x' = p\cos(135°) - q\sin(135°)$ $-1-\sqrt{2} = -p\cos45° - q\sin45°$ $-1-\sqrt{2} = -p\left(\frac{\sqrt{2}}{2}\right) - q\left(\frac{\sqrt{2}}{2}\right)$ $-1-\sqrt{2} = -\frac{\sqrt{2}}{2}p - \frac{\sqrt{2}}{2}q$.....(1) and $y' = y\cos(135°) + p\sin(135°)$ $1-\sqrt{2} = -q\cos45° + p\sin45°$ $1-\sqrt{2} = q\left(-\frac{\sqrt{2}}{2}\right) + p\left(\frac{\sqrt{2}}{2}\right)$ $-0,41 = -0,71q + 0,71p$.....(2) (1) + (2): $-2\sqrt{2} = -\sqrt{2}q$ $q = 2$ Substitute $q = 2$ into .....(1) $-2,41 = -0,71p - 0,71q$ (2) $1,42p = 2$ $p = 1,41$ $\therefore A = (\sqrt{2}\,;2)$ Note: If not left in surd form: 6/7	✓ equating ✓ substitution ✓ equating ✓ substitution $\frac{\sqrt{2}}{2}$ ✓ solving simultaneously ✓ answer for q ✓ answer for p (7)
	OR	

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	$-\frac{\sqrt{2}}{2}(p+q) = -1-\sqrt{2}$ $p+q = -\frac{2}{\sqrt{2}}(-1-\sqrt{2})$ $p+q = \sqrt{2}+2$ and $\frac{1}{\sqrt{2}}(p-q) = 1-\sqrt{2}$ $p-q = \sqrt{2}-2$ $p+q = \sqrt{2}+2$ $2p = 2\sqrt{2}$ $p = \sqrt{2}$ $q = 2$	✓ $-\frac{\sqrt{2}}{2}(p+q) = -1-\sqrt{2}$ ✓ substitution ✓ $\frac{1}{\sqrt{2}}(p-q) = 1-\sqrt{2}$ ✓ substitution $\frac{\sqrt{2}}{2}$ ✓ solving simultaneously ✓ answer for q ✓ answer for p (7)
	OR A$(p\,;q)$ is obtained from A$'$ by a rotation through 135° in a clockwise direction $p = (-1-\sqrt{2})\cos(-135°) - (1-\sqrt{2})\sin(-135°)$ $= (-1-\sqrt{2})\left(-\frac{1}{\sqrt{2}}\right) - (1-\sqrt{2})\left(-\frac{1}{\sqrt{2}}\right)$ $= \frac{2}{\sqrt{2}}$ $= \sqrt{2}$ $q = (1-\sqrt{2})\cos(-135°) + (-1-\sqrt{2})\sin(-135°)$ $= (1-\sqrt{2})\left(-\frac{1}{\sqrt{2}}\right) + (-1-\sqrt{2})\left(-\frac{1}{\sqrt{2}}\right)$ $= \frac{2\sqrt{2}}{\sqrt{2}}$ $= 2$ $\therefore A = (\sqrt{2}\,;2)$	✓ substituting $(-1-\sqrt{2})$ ✓ substitution $\frac{1}{\sqrt{2}}$ ✓ equating ✓ substitution $\frac{1}{\sqrt{2}}$ ✓ substituting $(-1-\sqrt{2})$ ✓ answer for q ✓ answer for p (7)

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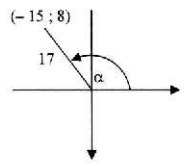
QUESTION 8

8.1	$\sin\alpha = \frac{8}{17}$ $\sin\alpha > 0$ $\therefore$ in second quadrant $y_\alpha = 8$ $r_\alpha = 17$ $x_\alpha = -15$ (Pythagoras) $\tan\alpha = -\frac{8}{15}$  	$x = -$✓15✓ ✓ answer (3) For drawing the radius vector in the correct quadrant 1/3 Without a sketch but correct values: 3/3
8.2	$\sin(90° + \alpha) = \cos\alpha$ $= -\frac{15}{17}$	✓ reduction ✓ answer (2) Answer only: full marks Cannot accept decimal values
8.3	$\cos2\alpha = 1 - 2\sin^2\alpha$ $= 1 - 2\left(\frac{8}{17}\right)^2$ $= \frac{161}{289}$ **OR** $\cos2\alpha = 2\cos^2\alpha - 1$ $= 2\left(\frac{-15}{17}\right)^2 - 1$ $= \frac{161}{289}$ **OR** $\cos2\alpha = \cos^2\alpha - \sin^2\alpha$ $= \left(\frac{-15}{17}\right)^2 - \left(\frac{8}{17}\right)^2$ $= \frac{161}{289}$	✓ expansion ✓ substitution ✓ any further calculation or answer (3) ✓ expansion ✓ substitution ✓ any further calculation or answer (3) ✓ expansion ✓ substitution ✓ any further calculation or answer (3) **[8]**

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QUESTION 9

NOTE: Only penalise once in the question for leaving out the x
Penalise once in this question for treating as an equation

9.1

$\sin(90°-x).\cos(180°-x)+\tan x.\cos(-x).\sin(180°+x)$
$=\cos x(-\cos x)+\tan x(\cos x)(-\sin x)$
$=-\cos^2 x-\frac{\sin x}{\cos x}\cos x\sin x$
$=-\cos^2 x-\sin^2 x$
$=-(\cos^2 x+\sin^2 x)$
$=-1$

✓ $\sin(90°-x)=\cos x$
✓ $\cos(180°-x)=-\cos x$
✓ $\cos(-x)=\cos x$
✓ $\sin(180°+x)=-\sin x$
✓ $\tan x=\frac{\sin x}{\cos x}$
✓ simplification
✓ answer
(7)

9.2

$\frac{\sin 190°\cos 225°\tan 390°}{\cos 100°\sin 135°}$
$=\frac{-\sin 10°(-\cos 45°)\tan 30°}{-\sin 10°\sin 45°}$
$=\frac{-\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{2}}}$ or $=-\tan 30°$
$=-\frac{1}{\sqrt{3}}$

If using $-\cos 80°$: no penalty

If the candidate stop at
$=\frac{-\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{2}}}$ 6/7

✓ $\sin 190°=-\sin 10°$
✓ $\cos 225°=-\cos 45°$
✓ $\tan 390°=\tan 30°$
✓ $\cos 100°=-\sin 10°$
✓ $\sin 135°=\sin 45°$ or $\cos 45°$
✓✓ substitution
(7)

9.3

$\sin x+2\cos^2 x=1$
$\sin x+2(1-\sin^2 x)=1$
$-2\sin^2 x+\sin x+1=0$
$2\sin^2 x-\sin x-1=0$
$(2\sin x+1)(\sin x-1)=0$
$\sin x=1$
$x=90°+k.360°;k\in Z$
Or

✓ substitution of identity
✓ standard form
✓ factorisation
✓ $\sin x=1;\sin x=-\frac{1}{2}$
✓ $x=90°+k.360°;k\in Z$
✓✓ answers (any two answers)
(7)
If $k\in Z$ not included: 6/7
Also $\pm\ k.360°;k\in N_0$ *or* Z

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$\sin x=-\frac{1}{2}$
$x=210°+k.360°;k\in Z$ *OR* $x=210°+k.360°$
or $x=330°+k.360°;k\in Z$ *or* $x=-30°+k.360°$
OR
$x=-150°+k.360°;k\in Z$ *OR* $x=-150°+k.360°;k\in Z$
or $x=330°+k.360°$ *or* $x=-30°+k.360°$

✓ manipulation
✓ substitution of identity
✓ co ratios
✓ $x=180°+(90°-2x)+k360°$
✓ $x=90°+k120°$
✓ $x=360°-(90°-2x)+k360°$
✓ $x=-270°-k360°$
(7)
If $k\in Z$ not included: 6/7

OR

$\sin x+2\cos^2 x=1$
$\sin x=1-2\cos^2 x$
$\sin x=-\cos 2x$
$\sin x=-[\sin(90°-2x)]$
$x=180°+(90°-2x)+k360°$ or $x=360°-(90°-2x)+k360°$
$3x=270°+k360°$ or $x=-270°-k360°$
$x=90°+k120°$
$k\in Z$

✓ manipulation
✓ substitution of identity
✓ co ratios

OR

$\sin x+2\cos^2 x=1$
$\sin x=1-2\cos^2 x$
$\sin x=-\cos 2x$
$-\cos(90°-x)=\cos 2x$
$2x=180°-(90°-x)+k360°$ or $2x=180°+(90°-x)+k360°$
$x=90°+k360°$ or $3x=270°+k360°$
$x=30°+k120°$
$k\in Z$

✓
$2x=180°-(90°-x)+k360°$
✓ $x=90°+k360°$
✓ $2x=180°+(90°-x)+k360°$
✓ $x=30°+k120°$
(7)
If $k\in Z$ not included: 6/7
[20]

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- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 10

10.1

$\frac{\sin(A+B)}{\cos(A+B)}=\frac{\sin A.\cos B+\cos A.\sin B}{\cos A.\cos B-\sin A.\sin B}$
$=\frac{\sin A.\cos B+\cos A.\sin B}{\cos A.\cos B-\sin A.\sin B}\times\frac{\frac{1}{\cos A.\cos B}}{\frac{1}{\cos A.\cos B}}$
$=\frac{\frac{\sin A.\cos B}{\cos A.\cos B}+\frac{\cos A.\sin B}{\cos A.\cos B}}{\frac{\cos A.\cos B}{\cos A.\cos B}-\frac{\sin A.\sin B}{\cos A.\cos B}}$
$=\frac{\tan A+\tan B}{1-\tan A.\tan B}$

✓ expansions
✓ divisions
✓ tanA and tanB
(3)

OR

RHS $=\frac{\tan A+\tan B}{1-\tan A.\tan B}$
$=\frac{\frac{\sin A}{\cos A}+\frac{\sin B}{\cos B}}{1-\frac{\sin A}{\cos A}\frac{\sin B}{\cos B}}\times\frac{\cos A.\cos B}{\cos A.\cos B}$
$=\frac{\sin A\cos B+\sin B\cos A}{\cos A\cos B-\sin A\sin B}$
$=\frac{\sin(A+B)}{\cos(A+B)}$
$=\tan(A+B)$
$=$ LHS

✓ $\frac{\sin A}{\cos A}$
✓ multiplication
✓ expansions
(3)

10.2

$\tan C=\tan(180°-(A+B))$
$\tan C=-\tan(A+B)$
$\tan C=-\left(\frac{\tan A+\tan B}{1-\tan A.\tan B}\right)$
$\tan C(1-\tan A.\tan B)=-(\tan A+\tan B)$
$\tan C-\tan A.\tan B.\tan C=-\tan A-\tan B$
$\tan A+\tan B+\tan C=\tan A.\tan B.\tan C$

OR

✓ C
✓ $-\tan(A+B)$
✓ substitution into formula
✓ multiplication with LCD
(4)
If no conclusion: 3/4

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

$\hat{C}=180°-(\hat{A}+\hat{B})$ (angles in a triangle)
$\tan C=\tan(180°-(A+B))$
$\tan C=\tan((180°-A)+(-B))$
$\tan C=\frac{\tan(180°-A)+\tan(-B)}{1-\tan(180°-A).\tan(-B)}$
$\tan C(1-\tan(180°-A).\tan(-B))=\tan(180°-A)+\tan(-B)$
$\tan C-\tan C\tan A\tan B=-\tan A-\tan B$
$\tan A+\tan B+\tan C=\tan A.\tan B.\tan C$

✓ C
✓ rearrange angle
✓ substitution into formula
✓ expansion
(4)

QUESTION 11

NOTE: Penalty of one for early rounding off once in this question

11.1.1

$B\hat{D}A=208°-67°$
$=141°$
$\frac{\sin D\hat{B}A}{97}=\frac{\sin 141°}{120}$
$\sin D\hat{B}A=0{,}5087006494...$
$D\hat{B}A=30{,}58°$
$\therefore$ Bearing of Ship A from Ship B
$=180°-(360°-208°)+30{,}58°$
$=58{,}58°$

✓ $B\hat{D}C=141°$
✓ sine rule
✓ substitution
✓ $\hat{B}=30{,}58°$
✓ method or $M\hat{B}D=28°$
✓ answer
(6)

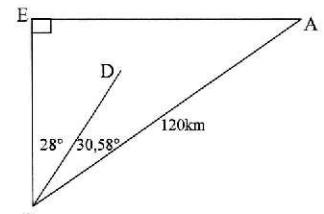
OR

$B\hat{D}A=208°-67°$
$=141°$
$\frac{\sin D\hat{B}A}{97}=\frac{\sin 141°}{120}$
$\sin D\hat{B}A=0{,}5087006494....$
$D\hat{B}A=30{,}58°$
then $360°-208°=N\hat{D}B$ (reflex angles)
$\therefore N\hat{D}B=152°$
but $M\hat{B}D+N\hat{D}B=180°$ (co - interior angles/ angles around a point)
$\therefore M\hat{B}D=28°$
then $M\hat{B}A=M\hat{B}D+D\hat{B}A$
$=30{,}58°+28°$
$=58{,}58°$

✓ $B\hat{D}C=141°$
✓ sine rule
✓ substitution
✓ $N\hat{D}B=152°$
✓ $M\hat{B}D=28°$
✓ answer
(6)

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

11.1.2 $\hat{B} = 30{,}58°$



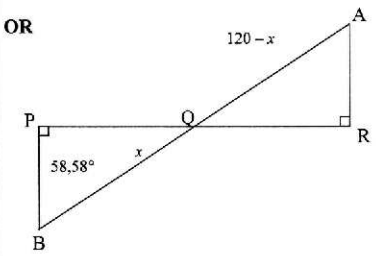
$\frac{EA}{120} = \sin(28° + 30{,}58°)$

$EA = 120\sin(28° + 30{,}58°)$

$EA = 102{,}4$ km

✓ definition
✓ substitution
✓ answer (3)

OR



Let BQ = x, then AQ = $120 - x$

$\sin 58{,}58° = \frac{PQ}{x}$ $\quad$ $\sin 58{,}58° = \frac{QR}{120 - x}$

$PQ = x.\sin 58{,}58°$ $\quad$ $QR = (120 - x)\sin 58{,}58°$

$PQ + QR = x.\sin 58{,}58° + (120 - x)\sin 58{,}58°$

$= 120\sin 58{,}58°$

$= 102{,}4$

✓ trigonometeric ratios
✓ sum
✓ answer (3)

OR

BP = AR (assume ships move at same speed)

✓ trigonometeric

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

$\Delta PBQ \equiv \Delta RAQ$ (angle, angle, side)

$\therefore BQ = QA = 60$ km

$\sin 58{,}58° = \frac{PQ}{60}$

$\therefore PQ = 60\sin 58{,}58°$

$= 51{,}20$ km

ratios
✓ 51,20 km
✓ answer (3)

$PR = 2PQ$

$= 102{,}4$ km

OR

$\frac{BM}{120} = \cos 31{,}42$

$BM = 120 \times \cos 31{,}42°$

$= 102{,}4$

✓ trigonometeric ratios
✓ substitution
✓ answer (3)

11.2 $AB = BC = a = c$

$b^2 = a^2 + c^2 - 2ac \times \cos B$

$b^2 = a^2 + a^2 - 2a \times a \times \cos B$

$b^2 = 2a^2 - 2a^2 \cos B$

$b^2 = 2a^2(1 - \cos B)$

$\frac{b^2}{2a^2} = 1 - \cos B$

$\cos B = 1 - \frac{b^2}{2a^2}$

✓ equal sides
✓ cos rule
✓ substitution
✓ simplification (4)

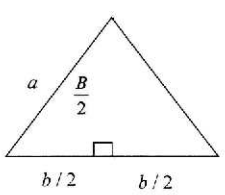
OR

$\sin\frac{B}{2} = \frac{b}{2a}$

$\cos B = 1 - 2\sin^2\frac{B}{2}$

$= 1 - 2\left(\frac{b}{2a}\right)^2$

$= 1 - \frac{b^2}{2a^2}$



✓ $\sin\frac{B}{2}$
✓ $\sin\frac{B}{2} = \frac{b}{2a}$
✓ formula
✓ substitution (4)
[13]

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

OR

$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

but $a = c$

$\cos B = \frac{a^2 + a^2 - b^2}{2a.a}$

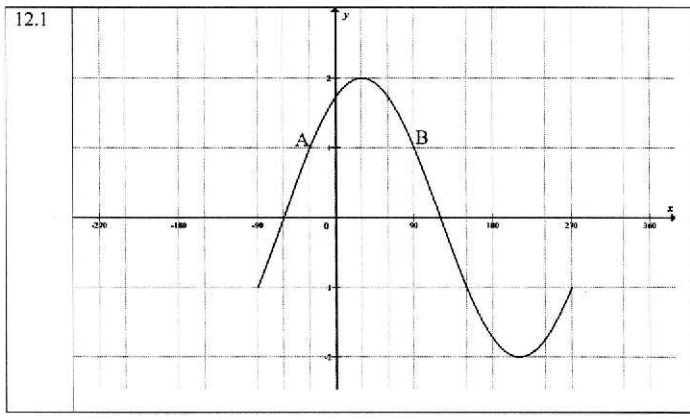
$= \frac{2a^2 - b^2}{2a^2}$

$= 1 - \frac{b^2}{2a^2}$

✓ cos rule
✓ equal sides
✓ substitution
✓ simplification (4)

QUESTION 12

12.1



✓ (120°; 0) or (–60°; 0)
✓ (30°; 2) or (210°; – 2) (2)

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 12

12.2 $\cos(x - 30°) = \frac{1}{2}$

$2\cos(x - 30°) = 1$

See points A and B on the graph

Note:

If drawn the line y = $\frac{1}{2}$ and put A and B on the graph: 0/2

If A and B on the x-axis: 1/2

If A = – 30° and B = 90° : 1/2

✓ manipulation
✓ answer (2)

A and B in the correct place on the graph: full marks

12.3 $\cos(x - 30°) = 0{,}5$

$x - 30° = 60°$ OR $x - 30° = -60°$

$x = 90°$ $\quad$ $x = -30°$

✓ 60° (ref angle)
✓ 90°
✓ – 30° (3)

Answer only: 3/3

12.4 $g'(x) = 0$ is at maximum and minimum values of graph

$x = 30°; 210°$

✓✓ one for each x-value (2)

12.5 $x \in [-90° ; -60°) \cup (120° ; 270°]$

OR

$-90° \le x < -60°$ or $120° < x \le 270°$

OR

If $x < -60°$ *or* $x > 120°$ 2/3

✓ notation
✓✓ critical values (3)

[12]

education

DEPARTMENT: EDUCATION
MPUMALANGA PROVINCE

Grade 12 Education Supplement 2010

Maths Paper 02
November 2008

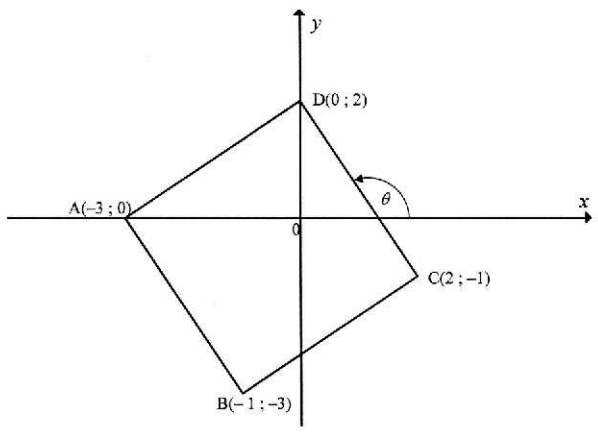
Mathematics

Maths Paper 02
November 2008

Together Educating the Nation

QUESTION 1

ABCD is a quadrilateral with vertices A(– 3 ; 0), B(– 1 ; – 3), C(2 ; –1) and D(0 ; 2).



1.1 Determine the coordinates of M, the midpoint of AC. (2)

1.2 Show that AC and BD bisect each other. (3)

1.3 Prove that $A\hat{D}C = 90°$. (4)

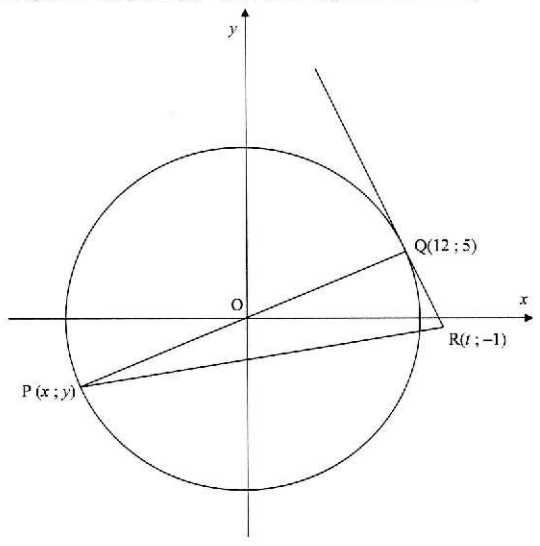
1.4 Show that ABCD is a square. (6)

1.5 Determine the size of θ, the angle of inclination of DC, correct to ONE decimal place. (3)

1.6 Does C lie inside or outside the circle with centre (0 ; 0) and radius 2? Justify your answer. (2)
[20]

QUESTION 2

O is the centre of the circle in the figure below. P(x ; y) and Q (12 ; 5) are two points on the circle. POQ is a straight line. The point R(t ; – 1) lies on the tangent to the circle at Q.



2.1 Determine the equation of the circle. (3)

2.2 Determine the equation of the straight line through P and Q. (2)

2.3 Determine x and y, the coordinates of P. (2)

2.4 Show that the gradient of QR is $-\frac{12}{5}$. (2)

2.5 Determine the equation of the tangent QR in the form $y = \ldots$ (3)

2.6 Calculate the value of t. (2)

2.7 Determine an equation of the circle with centre Q(12 ; 5) and passing through the origin. (3)
[17]

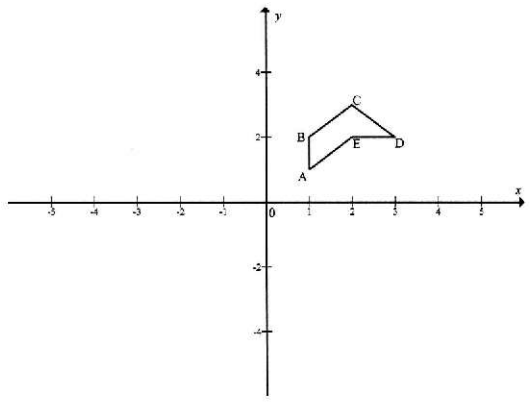
QUESTION 3

3.1 The point P($-\sqrt{2}$; $\sqrt{3}$) lies in a Cartesian plane. Determine the coordinates of the image of P if:

3.1.1 P is reflected in the line $y = x$. (2)

3.1.2 P is rotated about the origin through 180°. (2)

3.2 The vertices of the polygon ABCDE are shown in the grid. The coordinates are: A(1 ; 1), B(1 ; 2), C(2 ; 3), D(3 ; 2) and E(2 ; 2). Each of the points of ABCDE in the grid below is rotated about the origin in a clockwise direction through an angle of 90°.



3.2.1 Write down the coordinates of D′, the image of D. (1)

3.2.2 Sketch and label the vertices of A′B′C′D′E′, the image of ABCDE on DIAGRAM SHEET 1. (5)

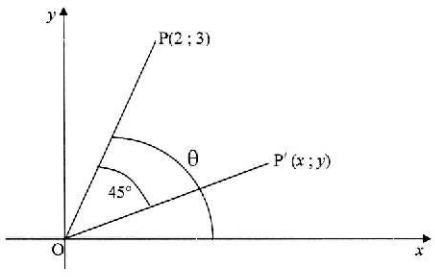
3.2.3 The polygon A′B′C′D′E′ is then enlarged through the origin by a factor of 3 in order to give the polygon A″B″C″D″E″. Write down the coordinates of D″, the image of D′. (2)

3.2.4 Write down the general transformation of a point (x ; y) in ABCDE to (x'' ; y'') after ABCDE has undergone the above two transformations; that is, rotation in a clockwise direction through an angle of 90° followed by an enlargement through the origin by a factor of 3. (4)

3.2.5 Calculate the ratio of Area ABCDE : Area A″B″C″D″E″. (2)
[18]

QUESTION 4

Determine the coordinates x and y of P′, the image of P(2 ; 3) when OP is rotated about the origin through an angle of 45° in the clockwise direction.



[7]

QUESTION 5

5.1 Do NOT use a calculator to answer this question. Show ALL calculations.

Prove that:

5.1.1 $\dfrac{\tan 480°.\sin 300°.\cos 14°.\sin(-135°)}{\sin 104°.\cos 225°} = \dfrac{3}{2}$. (6)

5.1.2 $\cos 75° = \dfrac{\sqrt{2}(\sqrt{3}-1)}{4}$ (4)

5.2 Prove that $\cos(90° - 2x).\tan(180° + x) + \sin^2(360° - x) = 3\sin^2 x$ (6)
[16]

QUESTION 6

6.1 6.1.1 Prove that $(\tan x - 1)(\sin 2x - 2\cos^2 x) = 2(1 - 2\sin x \cos x)$ (5)

6.1.2 Determine the general solution for: $\dfrac{\tan x - 1}{2} = -3$ correct to ONE decimal place. (5)

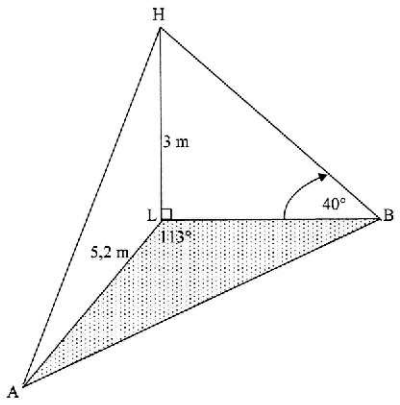
6.2 If $\cos\beta = \dfrac{p}{\sqrt{5}}$ where $p < 0$ and $\beta \in [180° ; 360°]$, determine, using a diagram, an expression in terms of p for:

6.2.1 $\tan\beta$ (4)

6.2.2 $\cos 2\beta$ (3)
[17]

QUESTION 7

A, B and L are points in the same horizontal plane, HL is a vertical pole of length 3 metres, AL = 5,2 m, the angle $A\hat{L}B = 113°$ and the angle of elevation of H from B is 40°.



7.1 Calculate the length of LB. (2)

7.2 Hence, or otherwise, calculate the length of AB. (4)

7.3 Determine the area of ΔABL. (4)
[10]

QUESTION 8

Consider the functions $f(x) = \cos 3x$ and $g(x) = \sin x$ for $x \in [-90° ; 180°]$.

8.1 Solve for x if $f(x) = g(x)$. (8)

8.2 Sketch the graphs of f and g on the system of axes on DIAGRAM SHEET 2 for $x \in [-90° ; 180°]$. (6)

8.3 Solve for x if $f(x) \leq g(x)$ where $x \in [-90° ; 0°]$. (3)
[17]

QUESTION 9

The time taken, in minutes, to complete a 5 kilometre race by a group of 10 runners is given below:

18 21 16 24 28 20 22 29 19 23

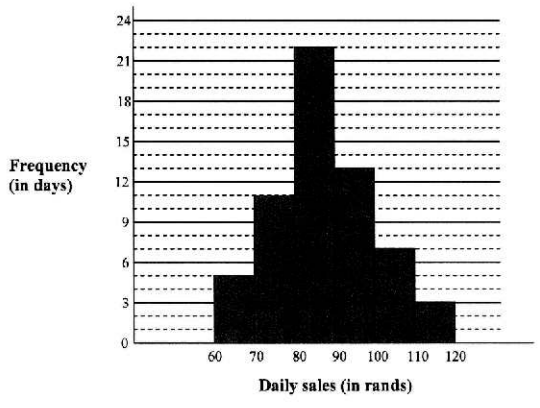
9.1 Calculate the mean time taken to complete the race. (2)

9.2 Calculate the standard deviation of the time taken to complete the race. (Use the formula on the information sheet.) (4)

9.3 How many runners completed the race within one standard deviation of the mean? (2)
[8]

QUESTION 10

A street vendor has kept a record of sales for November and December 2007.
The daily sales in rands is shown in the histogram below.



10.1 On DIAGRAM SHEET 3, complete the cumulative frequency table for the sales over November and December. (3)

10.2 Draw an ogive for the sales over November and December on DIAGRAM SHEET 3. (3)

10.3 Use your ogive to determine the median value for the daily sales. Explain how you obtain your answer. (1)

10.4 Estimate the interval of the upper 25% of the daily sales. (2)
[9]

QUESTION 11

A parachutist jumps out of a helicopter and his height above ground level is estimated at various times after he opened his parachute. The following table gives the results of the observations where y measures his height above ground level in metres and t represents the time in seconds after he opened his parachute.

t	2	3	4	5	6	7	8
y	500	300	200	120	70	40	20

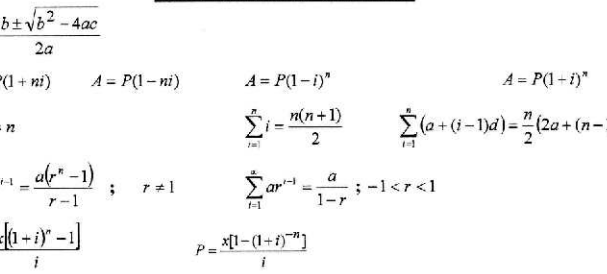
11.1 On DIAGRAM SHEET 4, draw a scatter plot for the above information. (2)

11.2 Describe the curve of best fit. (1)

11.3 Use the scatter plot to estimate the height of the parachutist 5,5 seconds after he had opened his parachute. (1)
[4]

QUESTION 12

The box and whisker plots below summarise the final test scores for two of Mr Jack's Mathematics classes from the same grade.



12.1 Describe the features in the scores that are the same for both classes. (2)

12.2 Calculate the interquartile range for Class B. (2)

12.3 Mr Jack considers the median of each class and reports that there is no significant difference in the performance between them. Is Mr Jack's conclusion valid? Support your answer with reasons. (3)
[7]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS
INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} \left(a + (i-1)d\right) = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$\sum_{i=1}^{n} ar^{i-1} = \frac{a(r^n - 1)}{r-1} \; ; \quad r \neq 1 \qquad \sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1-r} \; ; \; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A \qquad \text{area } \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}$$

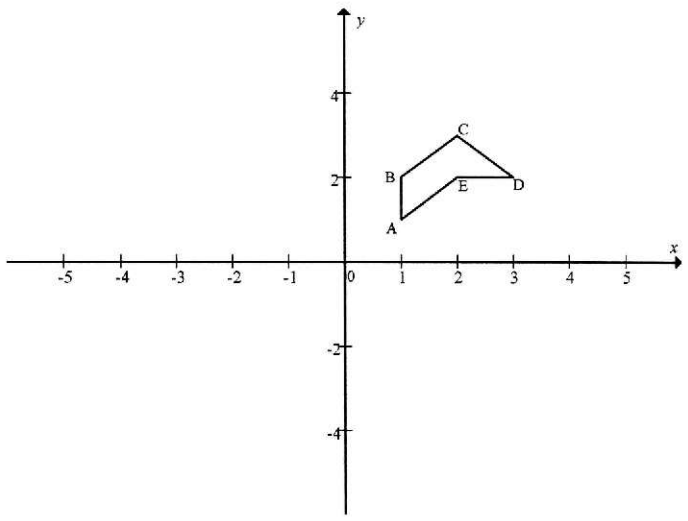
$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

EXAMINATION NUMBER:

DIAGRAM SHEET 1

QUESTION 3.2



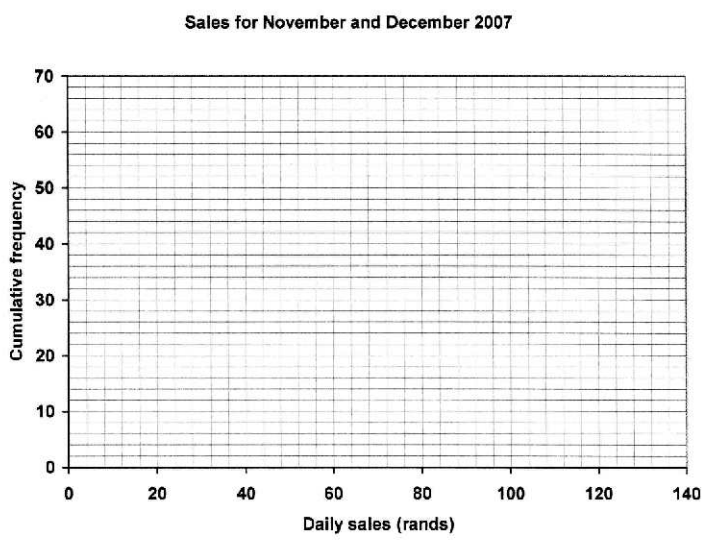
EXAMINATION NUMBER:

DIAGRAM SHEET 3

QUESTION 10.1

DAILY SALES	FREQUENCY	CUMULATIVE FREQUENCY

QUESTION 10.2



EXAMINATION NUMBER:

DIAGRAM SHEET 2

QUESTION 8.2

EXAMINATION NUMBER:

DIAGRAM SHEET 4

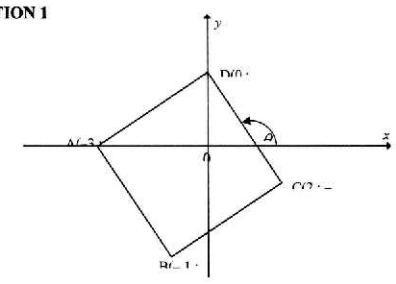
QUESTION 11.1

Exercise

Exercise

- **Continued accuracy applies as a rule in the memorandum.**
- **If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.**

QUESTION 1



1.1	M $\left(\frac{2-3}{2};\frac{-1+0}{2}\right)$ $=\left(-\frac{1}{2};-\frac{1}{2}\right)$	✓ substitution into midpoint formula ✓ answer for both coordinates (2) Answer only: 1 mark per coordinate Wrong formula: 0 / 2
1.2	Midpoint BD $=\left(\frac{-1+0}{2};\frac{-3+2}{2}\right)$ $=\left(-\frac{1}{2};-\frac{1}{2}\right)$ ∴ Midpoint of AC and BD are the same point therefore AC and BD bisect each other **OR**	✓ substitution into formula ✓ answer ✓ conclusion (midpoints are the same) (3)

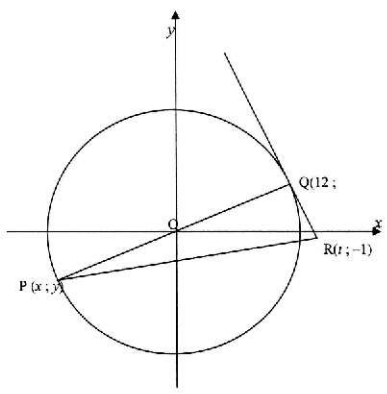
	$AM=\sqrt{\left(-3+\frac{1}{2}\right)^2+\left(0+\frac{1}{2}\right)^2}$ $AM=\sqrt{6,5}$ $CM=\sqrt{\left(2+\frac{1}{2}\right)^2-\left(-1+\frac{1}{2}\right)^2}$ $CM=\sqrt{6,5}$ $BM=\sqrt{\left(-1+\frac{1}{2}\right)^2+\left(-3+\frac{1}{2}\right)^2}$ $BM=\sqrt{6,5}$ $DM=\sqrt{\left(0+\frac{1}{2}\right)^2+\left(2+\frac{1}{2}\right)^2}$ $DM=\sqrt{6,5}$ AC and BD bisect each other	2 / 3 for answer on the left (because candidate did not show that M is on BD)
1.3	$m_{AD}=\frac{2-0}{0+3}$ $m_{AD}=\frac{2}{3}$ $m_{CD}=\frac{-1-2}{2-0}$ $m_{CD}=-\frac{3}{2}$ $m_{AD}\times m_{CD}$ $=\frac{2}{3}\times-\frac{3}{2}$ $=-1$ $\therefore AD\perp CD$ $\therefore A\hat{D}C=90°$ Note: If do: $m_{AD}\times m_{CD}=-1$, $\frac{2}{3}\times-\frac{3}{2}=-1$, $-1=-1$ then **3 / 4** if calculated the gradients correctly. If $m_{AD}\times m_{CD}=-1$ and conclude AD ⊥ CD without any working, then **1 / 4** **OR** $\tan\theta=m_{CD}$ $\tan\theta=-\frac{3}{2}$ $\theta=123,69°$ $\tan D\hat{A}C=\frac{2}{3}$ $D\hat{A}C=33,69°$ $A\hat{D}C=123,69°-33,69°$ $A\hat{D}C=90°$	✓ answer m_{AD} ✓ answer m_{CD} ✓ $m_{AD}\times m_{CD}=-1$ ✓ conclude $A\hat{D}C=90°$ (4) ✓ $\tan\theta=m_{CD}$ ✓ $\theta=123.69°$ ✓ $D\hat{A}C=33,69°$ ✓ $A\hat{D}C=90°$ (4)

	OR $AD^2=(2-0)^2+(0-(-3))^2$ $AD^2=13$ $DC^2=(2-(-1))^2+(0-2)^2$ $DC^2=13$ $AC^2=(0-(-1))^2+(-3-2)^2$ $AC^2=26$ AD^2+DC^2 $=13+13$ $=26$ $=AC^2$ $\therefore$ AD ⊥ DC $\therefore A\hat{D}C=90°$	✓ $AD^2=13$ ✓ $DC^2=13$ ✓ $AC^2=26$ ✓ conclusion (4)
1.4	$BD=\sqrt{(2+3)^2+(0+1)^2}$ $=\sqrt{26}$ $AC=\sqrt{(-3-2)^2+(0+1)^2}$ $=\sqrt{26}$ diagonals are equal diagonals bisect each other (Proved in 1.2) (i.e. ABCD is a rectangle) $m_{AC}.m_{BD}$ $=\frac{1}{-5}\times\frac{5}{1}$ $=-1$ AC ⊥ BD **OR** $AD^2=(2-0)^2+(0-(-3))^2$ $AD^2=13$ $DC^2=(2-(-1))^2+(0-2)^2$ $DC^2=13$ The figure is a rectangle and one pair of adjacent sides are equal in length ∴ it is a square. **OR**	✓ answer for BD ✓ answer for AC ✓ diagonals are equal ✓ bisect each other ✓ $m_{AC}.m_{BD}=-1$ ✓ . AC ⊥ BD (6) ✓ substitution ✓ answer for AD ✓ substitution ✓ answer for DC ✓✓ conclusion (6)

	$AD^2=(2-0)^2+(0-(-3))^2$ $AD^2=13$ $DC^2=(2-(-1))^2+(0-2)^2$ $DC^2=13$ $AB^2=(-3-(-1))^2-(0-(-3))^2$ $AB^2=13$ $BC^2=(2-(-1))^2+(-1-(-3))^2$ $BC^2=13$ All four sides equal and one internal angle equal to 90° **OR** The diagonals bisect one another $A\hat{D}C=90°$ $AD^2=(2-0)^2+(0-(-3))^2$ $AD^2=13$ $DC^2=(2-(-1))^2+(0-2)^2$ $DC^2=13$ ∴ adjacent sides equal in length ∴ ABCD is a square	✓ answer for AD ✓ answer for AB ✓ answer for DC ✓ answer for BC ✓ all four sides are equal ✓ one internal angle equal to 90° (6) ✓ diagonals bisect each other ✓ $A\hat{D}C=90°$ ✓ substitution into distance formula ✓ answer for AD ✓ answer for DC ✓ conclusion (6)
1.5	$\tan\theta=\frac{2+1}{0-2}$ $\tan\theta=-\frac{3}{2}$ $\theta=-56,30993247\ldots+180°$ $\theta=123,7°$ **OR** $\tan D\hat{A}O=\frac{2}{3}$ $D\hat{A}O=33,7°$ $A\hat{D}C=90°$ $\theta=90°+33,7°$ $\theta=123,7°$ Penalty 1 for incorrect rounding	✓ gradient of CD ✓ $\tan\theta=-\frac{3}{2}$ ✓ answer (3) ✓ $\theta=90°+D\hat{A}O$ ✓ $\tan D\hat{A}O=\frac{2}{3}$ ✓ answer (3)

1.6	$OC^2 = (2-0)^2 + (-1-0)^2$ $OC^2 = 5$ $OC = 2{,}236067977$ $OC > 2$ C lies outside the circle OR $OC^2 = (2-0)^2 + (-1-0)^2$ $OC^2 = 5$ $OC^2 > 4$ C lies outside the circle OR $x^2 - y^2 = 4$ $(2)^2 - (-1)^2 = 5 > 4$ C lies outside the circle	✓ OC^2 ✓ answer (2) Answer only: 0 / 2 **[20]**

QUESTION 2

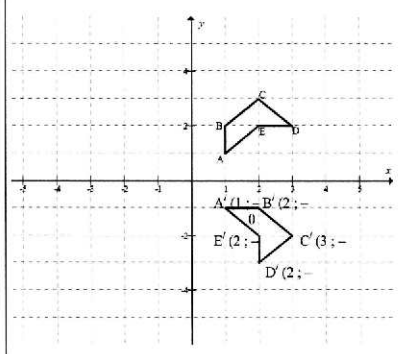


2.1	$r^2 = OQ^2$ $= (5)^2 + (12)^2$ $= 169$ $\therefore x^2 + y^2 = 169$ OR $x^2 + y^2 = (5)^2 + (12)^2 = 169$	✓ substituting (5 ; 12) into $x^2 + y^2$ ✓ 169 ✓ $x^2 + y^2 = 169$ (3) ✓ $x^2 + y^2 = r^2$ ✓ substitution coordinates ✓ 169 (3) Answer only: Full marks
2.2	$m_{PQ} = \frac{5-0}{12-0)}$ $m_{PQ} = \frac{5}{12}$ $\therefore y = \frac{5}{12}x$	✓ gradient ✓ $c = 0$ (2)

2.3	P(− 12; − 5) (By symmetry) **OR** $x^2 + y^2 = 169$ $x^2 - \left(\frac{5}{12}x\right)^2 = 169$ $144x^2 + 25x^2 = 169 \times 144 = 24336$ $169x^2 = 24336$ $x^2 = 144$ $x = \pm 12$ $x = -12$ $y = -5$	✓ $x = -12$ ✓ $y = -5$ (2)
2.4	tangent ⊥ diameter $m_{PQ} \times m_{QR} = -1$ $m_{PQ} = \frac{5}{12}$ $\therefore m_{QR} = -\frac{1}{\frac{5}{12}} = -\frac{12}{5}$ **OR** PQ ⊥ QR $m_{QR} = -\frac{12}{5}$	✓✓ $m_{PQ} \times m_{QR} = -1$ (2) ✓✓ PQ ⊥ QR (2)
2.5	$y = \frac{-12}{5}x - c$ $5 = \frac{-12}{5}(12) + c$ $c = \frac{169}{5}$ $y = -\frac{12}{5}x + \frac{169}{5}$ OR $y = -2{,}4x + 33{,}8$ **OR**	✓ $y = mx + c$ ✓substitution of gradient and (12 ; 5) ✓calculation of c. (3)

	$y - y_1 = m(x - x_1)$ $y - 5 = -\frac{12}{5}(x - 12)$ $5y - 25 = -12(x - 12)$ $5y = -12x + 144 + 25$ $5y = -12x + 169$ $12x + 5y - 169 = 0$ $y = -\frac{12}{5}x + \frac{169}{5}$	✓ formula ✓ substitution of gradient and (12 ; 5) ✓equation in correct form (3)
2.6	$-1 = \frac{-12}{5}(t) + \frac{169}{5}$ $12t = 174$ $t = \frac{174}{12}$ $t = 14{,}5$ **OR** $m_{QQ} \times m_{QR} = -1$ $\frac{5}{12} \times \frac{-6}{t-12} = -1$ $t = 14{,}5$ **OR** $PQ^2 + QR^2 = PR^2$ $576 + 100 + (12-t)^2 - 36 = (t+12)^2 + 16$ $712 + 144 - 24t + t^2 = t^2 + 24t + 144 + 16$ $-48t = -696$ $t = 14{,}5$	✓substitution of $(t ; -1)$ ✓answer (2) ✓ $\frac{5}{12} \times \frac{-6}{t-12} = -1$ ✓ answer (2) ✓ Pythagoras with substitution ✓ answer (2)
2.7	$(x-12)^2 + (y-5)^2 = OQ^2$ $OQ^2 = (12-0)^2 + (5-0)^2 = 169$ $(x-12)^2 + (y-5)^2 = 169$ **OR** $(x)^2 + (y)^2 = 169$ By translating 12 units right and 5 units up $(x-12)^2 + (y-5)^2 = 169$	✓ $(x-12)^2$ ✓ $(y-5)^2$ ✓169 (3) If answer only: $(x-12)^2 + (y-5)^2 = 169$: 3 / 3 **[17]**

QUESTION 3

3.1.1	$P'(\sqrt{3};-\sqrt{2})$	✓ x coordinate of P' ✓ y-coordinate of P' (2)
3.1.2	$P'(\sqrt{2},-\sqrt{3})$	✓ x coordinate of P' ✓ y-coordinate of P' (2)
3.2.1	$D'(2;-3)$ If rotated anti-clockwise: $D'(-2;3)$	✓ answer (1) No mark for $D'(-2;3)$
3.2.2		✓ coordinates A' ✓ coordinates B' ✓ coordinates C' ✓ coordinates E' ✓ rotation correct (5) If all the points on the sketch are correct and labels are A' etc: 5 / 5 If all the points on the sketch are correct and labels at incorrect point: 4 / 5 Deduct 2 marks for anti-clockwise direction If write down coordinates correctly and did not sketch: 4 / 5
3.2.3	$D''(6;-9)$ If rotated anti-clockwise: $D''(-6;9)$	✓ x-coordinate ✓ y-coordinate (2)
3.2.4	$(x;y)\rightarrow(y;-x)$ $(y;-x)\rightarrow(3y;-3x)$ $\therefore(x;y)\rightarrow(3y;-3x)$	✓✓ $(y;-x)$ ✓✓ $(3y;-3x)$ (4) Answer only: 4 / 4 If answer $(ky;-kx)$ 3 / 4 If Answer: $3(y;-x)$

	If rotated anti-clockwise the answer would be: $(x;y)\rightarrow(-y;x)$ $(y;-x)\rightarrow(-3y;3x)$ $\therefore(x;y)\rightarrow(-3y;3x)$	4 / 4
3.2.5	Area ABCDE : area $A''B''C''D''E''$ $=1^2:3^2$ $=1:9$ **OR** $\frac{ABCDE}{A''B''C''D''E''}$ $=\frac{1}{9}$	✓✓ answer (2) If $\frac{A''B''C''D''E''}{ABCDE}$ $=\frac{9}{1}$ 0 / 2

[18]

QUESTION 4

	$x'=x\cos(-45°)-y\sin(-45°)$ $x'=2\cos45°+3\sin45°$ $x'=2\left(\frac{\sqrt{2}}{2}\right)+3\left(\frac{\sqrt{2}}{2}\right)$ $x'=\frac{5\sqrt{2}}{2}$ *or* $x'=\frac{5}{\sqrt{2}}$ $x'=3,54$ and $y'=y\cos(-45°)+x\sin(-45°)$ $y'=3\cos45°-2\sin45°$ $y'=3\left(\frac{\sqrt{2}}{2}\right)-2\left(\frac{\sqrt{2}}{2}\right)$ $y'=\frac{\sqrt{2}}{2}$ *or* $\frac{1}{\sqrt{2}}$ *or* 0,71 $P'\left(\frac{5\sqrt{2}}{2};\frac{\sqrt{2}}{2}\right)$ A penalty of 2 marks for substituting 45° instead of $-45°$. The answer will then be $\left(-\frac{\sqrt{2}}{2};\frac{5\sqrt{2}}{2}\right)$ or $(-0,71;3,54)$	✓ formula ✓ $-45°$ or $315°$ ✓ substitution of $\left(\frac{\sqrt{2}}{2}\right)$ or $\left(\frac{1}{\sqrt{2}}\right)$ ✓ answer for x ✓ formula ✓ substitution of $\left(\frac{\sqrt{2}}{2}\right)$ ✓ answer for y (7)

	OR If a candidate rotates clockwise and substitutes 45° the formulae will be: $x'=x\cos\theta+y\sin\theta$ $x'=2\cos45°+3\sin45°$ $x'=2\left(\frac{\sqrt{2}}{2}\right)+3\left(\frac{\sqrt{2}}{2}\right)$ $x'=3,54$ $y'=y\cos\theta-x\sin\theta$ $y'=3\cos45°-2\sin45°$ $y'=3\left(\frac{\sqrt{2}}{2}\right)-2\left(\frac{\sqrt{2}}{2}\right)$ $y'=0,71$	✓ formula for x' ✓ 45° ✓ substitution ✓ answer for x' ✓ formula for y' ✓ substitution ✓ answer for y' (7)
	OR Let OP = OP′ = $r=\sqrt{13}$ The x-coordinate of P = $r\cos(\theta-45°)$ $x'=r(\cos\theta.\cos45°+\sin\theta.\sin45°)$ $x'=\sqrt{13}\cos\theta.\cos45°+\sqrt{13}\sin\theta.\sin45°$ $x'=\sqrt{13}.\frac{2}{\sqrt{13}}.\frac{\sqrt{2}}{2}+\sqrt{13}.\frac{3}{\sqrt{13}}.\frac{\sqrt{2}}{2}$ $x'=\sqrt{2}+\frac{3\sqrt{2}}{2}$ $x'=\frac{5\sqrt{2}}{2}$ The y-coordinate of P = $r\sin(\theta-45°)$ $y'=r(\sin\theta.\cos45°-\cos\theta.\sin45°)$ $y'=\sqrt{13}\sin\theta.\cos45°-\sqrt{13}\cos\theta.\sin45°$ $y'=\sqrt{13}.\frac{3}{\sqrt{13}}.\frac{\sqrt{2}}{2}-\sqrt{13}.\frac{2}{\sqrt{13}}.\frac{\sqrt{2}}{2}$ $y'=\frac{3\sqrt{2}}{2}-\sqrt{2}$ $y'=\frac{\sqrt{2}}{2}$ $P'\left(\frac{5\sqrt{2}}{2};\frac{\sqrt{2}}{2}\right)$	✓ formula $r\cos(\theta-45°)$ ✓ expansion ✓ substitution ✓ answer for x ✓ formula $r\sin(\theta-45°)$ ✓ expansion ✓ answer for y (7)
	OR	

	$2=x\cos45°-y\sin45°$ $3=y\cos45°+x\sin45°$ $2=\frac{1}{\sqrt{2}}x-\frac{1}{\sqrt{2}}y$ $\times\sqrt{2}:2\sqrt{2}=x-y$ --- (1) $3=\frac{1}{\sqrt{2}}y+\frac{1}{\sqrt{2}}x$ $\times\sqrt{2}:3\sqrt{2}=x+y$ --- (2) (1) + (2) $2x=5\sqrt{2}$ $x=\frac{5\sqrt{2}}{2}$ $\therefore3\sqrt{2}=\frac{5\sqrt{2}}{2}+y$ $\therefore y=\frac{1}{2}\sqrt{2}$	✓ formula ✓ formula ✓ substitution ✓ substitution ✓ solving simultaneous ✓ answer x ✓ answer y (7)
	OR $(x';y')=(r\cos(\theta-45°);r\sin(\theta-45°))$ $x^2-y^2=r^2$ $2^2-3^2=r^2$ $r=\sqrt{13}$ $\tan\theta=\frac{3}{2}$ $\theta=56,30993247...°$ $x'=r\cos(\theta-45°)$ $x'=\sqrt{13}\cos(56,3...°-45°)$ $x'=3,54$ $y'=r\sin(\theta-45°)$ $y'=\sqrt{13}\sin(56,3...°-45°)$ $y'=0,71$	✓ $r=\sqrt{13}$ ✓ $\tan\theta=\frac{3}{2}$ ✓ $\theta=56,30993247...°$ ✓ $x'=r\cos(\theta-45°)$ ✓ $x'=3,54$ ✓ $y'=r\sin(\theta-45°)$ ✓ $y'=0,71$ (7) Answer only: 6 / 7

[7]

QUESTION 5

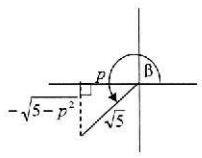
Penalise 1 mark for treating as an equation in this question.

5.1.1	$\frac{\tan 480°.\sin 300°.\cos 14°.\sin(-135°)}{\sin 104°.\cos 225°}$ $=\frac{\tan 120°.(-\sin 60°).\cos 14°.(-\sin 45°)}{\sin 76°.(-\cos 45°)}$ $=\frac{(-\tan 60°).(-\sin 60°).\cos 14°.(-\sin 45°)}{\cos 14°.(-\cos 45°)}$ $=\frac{\left(-\sqrt{3}\right)\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)}{\left(-\frac{\sqrt{2}}{2}\right)}$ $=\frac{3}{2}$ **OR** $\frac{\tan 480°.\sin 300°.\cos 14°.\sin(-135°)}{\sin 104°.\cos 225°}$ $=\frac{\tan 120°.(-\sin 60°).\cos 14°.(-\sin 45°)}{\sin 76°.(-\cos 45°)}$ $=\frac{(-\tan 60°).(-\sin 60°).\sin 76°.\tan 45°}{\sin 76°}$ $=\left(-\sqrt{3}\right)\left(-\frac{\sqrt{3}}{2}\right).1$ $=\frac{3}{2}$	✓ $-\sin 60°$ ✓ $-\sin 45°$ ✓ $-\cos 45°$ ✓ $-\tan 60°$ ✓ $\cos 14°$ or $\sin 76°$ ✓ substitution Penalise 1 mark for treating as an equation in this question. (6) ✓ $-\sin 60°$ ✓ $-\sin 45°$ ✓ $-\cos 45°$ ✓ $-\tan 60°$ ✓ $\sin 76°$ ✓ substitution (6)
5.1.2	$\cos 75°$ $=\cos(45°+30°)$ $=\cos 45°.\cos 30°-\sin 45°.\sin 30°$ $=\frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{2}.\frac{1}{2}$ $=\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}$ $=\frac{\sqrt{2}.\sqrt{3}-\sqrt{2}}{4}$ $=\frac{\sqrt{2}(\sqrt{3}-1)}{4}$ **OR**	✓ $\cos(45°+30°)$ ✓ expansion ✓ substitution ✓ simplification (4)

	$\cos 75°$ $=\cos(45°+30°)$ $=\cos 45°.\cos 30°-\sin 45°.\sin 30°$ $=\frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}.\frac{1}{2}$ $=\frac{\sqrt{3}}{2\sqrt{2}}-\frac{1}{2\sqrt{2}}$ $=\frac{\sqrt{3}-1}{2\sqrt{2}}$	✓ $\cos(45°+30°)$ ✓ expansion ✓ substitution ✓ simplification (4)
5.2	$\cos(90°-2x).\tan(180°-x)-\sin^2(360°-x)$ $=\sin 2x.\tan x+\sin^2 x$ $=2\sin x.\cos x.\frac{\sin x}{\cos x}+\sin^2 x$ $=2\sin^2 x+\sin^2 x$ $=3\sin^2 x$	✓ $\sin 2x$ ✓ $\tan x$ ✓ $\sin^2 x$ ✓ $\tan x=\frac{\sin x}{\cos x}$ ✓ $\sin 2x=2\sin x.\cos x$ ✓ $2\sin^2 x$ (6) If uses $\cos 2x$ instead of $\sin 2x$ and then works correctly: max 3/6 **[16]**

QUESTION 6

6.1.1	$(\tan x-1)(\sin 2x-2\cos^2 x)$ $=\left(\frac{\sin x}{\cos x}-1\right)(2\sin x.\cos x-2\cos^2 x)$ $=\left(\frac{\sin x}{\cos x}-1\right)2\cos x(\sin x-\cos x)$ $=2(\sin x-\cos x)^2$ $=2(\sin^2 x-2\sin x.\cos x+\cos^2 x)$ $=2(1-2\sin x.\cos x)$ **OR** $(\tan x-1)(\sin 2x-2\cos^2 x)$ $=\left(\frac{\sin x}{\cos x}-1\right)(2\sin x.\cos x-2\cos^2 x)$ $=2\sin^2 x-2\sin x.\cos x-2\sin x.\cos x+2\cos^2 x$ $=2(\sin^2 x-2\sin x\cos x+\cos^2 x)$ $=2(1-2\sin x.\cos x)$ **OR** $2(1-2\sin x\cos x)$ $=2(\sin^2 x+\cos^2 x-2\sin x\cos x)$ $=2(\sin x-\cos x)^2$ $=2\cos^2 x(\frac{\sin x}{\cos x}-1)^2$ $=2\cos^2 x(\tan x-1)(\tan x-1)$ $=(2\cos^2 x.\tan x-2\cos^2 x)(\tan x-1)$ $=(2\sin x\cos x-2\cos^2 x)(\tan x-1)$ $=(\sin 2x-2\cos^2 x)(\tan x-1)$ **OR** $LHS=(\tan x-1)(\sin 2x-\cos^2 x)$ $=\frac{\sin x-\cos x}{\cos x}(2\sin x.\cos x-\cos^2 x)$ $=2(\sin x-\cos x)^2$ $RHS=2(\sin^2 x+\cos^2 x-2\sin x\cos x)$ $=2(\sin x-\cos x)^2$ $=LHS$	✓ $\frac{\sin x}{\cos x}=\tan x$ ✓ $\sin 2x=2\sin x.\cos x$ ✓ factorisation ✓ simplification ✓ $\sin^2 x+\cos^2 x=1$ (5) ✓ $\frac{\sin x}{\cos x}=\tan x$ ✓ $\sin 2x=2\sin x.\cos x$ ✓ simplification ✓ factorisation ✓ $\sin^2 x+\cos^2 x=1$ ✓ $\sin^2 x+\cos^2 x=1$ ✓ factorisation ✓ $\frac{\sin x}{\cos x}=\tan x$ ✓ $\sin 2x=2\sin x.\cos x$ ✓ simplification ✓ $\frac{\sin x}{\cos x}=\tan x$ ✓ simplification ✓ $\sin 2x=2\sin x.\cos x$ ✓ $\sin^2 x-\cos^2 x=1$ ✓ factorisation

6.1.2	$\frac{\tan x-1}{2}=-3$ $\tan x-1=-6$ $\tan x=-5$ $x=-78{,}7°+k.180°$ $k\in Z$ **OR** $\frac{\tan x-1}{2}=-3$ $\tan x-1=-6$ $\tan x=-5$ $x=101{,}3°+k.180°$ $k\in Z$ **OR** $\frac{\tan x-1}{2}=-3$ $\tan x-1=-6$ $\tan x=-5$ $x=101{,}3°+k.360°$ *or* $x=281{,}3°+k.360°$ $k\in Z$ **OR** If the candidate has used $\tan(x-1)=-6$ max of 2 / 5	✓ simplification ✓ simplification ✓ $-78{,}7°$ ✓ $+k.180°$ ✓ $k\in Z$ (5)
6.2.1	$\cos\beta=\frac{p}{\sqrt{5}}$ $x=p$ $r=\sqrt{5}$ $y=-\sqrt{5-p^2}$ $\therefore\tan\beta=\frac{-\sqrt{5-p^2}}{p}$  	✓ third quadrant ✓ $y=-\sqrt{5-p^2}$ ✓✓ answer (4) If p is negative:3/4

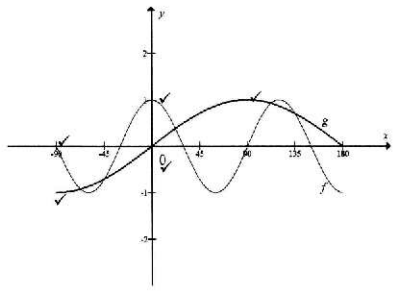
6.2.2	$\cos 2\beta = 2\cos^2\beta - 1$ $= 2\left(\frac{p}{\sqrt{5}}\right)^2 - 1$ $= \frac{2p^2}{5} - 1$	✓ $2\cos^2\beta - 1$ ✓✓ $2\left(\frac{p}{\sqrt{5}}\right)^2 - 1$ or $\frac{2p^2}{5} - 1$ (3)
	OR	
	$\cos 2\beta = 1 - 2\sin^2\beta$ $= 1 - 2\left(\frac{-\sqrt{5-p^2}}{\sqrt{5}}\right)^2$ $= 1 - \frac{2(5-p^2)}{5}$ $= \frac{2p^2-5}{5}$	✓ $1 - 2\sin^2\beta$ ✓✓ $1 - 2\left(\frac{-\sqrt{5-p^2}}{\sqrt{5}}\right)^2$ or $1 - \frac{2(5-p^2)}{5}$ or $\frac{2p^2-5}{5}$ (3)
	OR	
	$\cos 2\beta = \cos^2\beta - \sin^2\beta$ $= \left(\frac{p}{\sqrt{5}}\right)^2 - \left(\frac{-\sqrt{5-p^2}}{\sqrt{5}}\right)^2$ $= \frac{p^2}{5} - \frac{5-p^2}{5}$ $= \frac{2p^2-5}{5}$	✓ $\cos^2\beta - \sin^2\beta$ ✓ $\left(\frac{p}{\sqrt{5}}\right)^2$ ✓ $\left(\frac{-\sqrt{5-p^2}}{\sqrt{5}}\right)^2$ or $\frac{p^2}{5} - \frac{5-p^2}{5}$ or $\frac{2p^2-5}{5}$ (3) **[17]**

QUESTION 7

7.1	$\frac{3}{LB} = \tan 40°$ $LB = \frac{3}{\tan 40°}$ $LB = 3{,}58$ m (3,5752.....) (3,5 m ; 3,57 m ; 3,6 m)	✓ trig ratio ✓ answer (2)
	OR	
	$\frac{LB}{\sin 50°} = \frac{3}{\sin 40°}$ $LB = \frac{3\sin 50°}{\sin 40°}$ $LB = 3{,}58$ m (3,5752.....)	✓ sine rule ✓ answer (2)
7.2	$AB^2 = AL^2 + BL^2 - 2.AL.BL.\cos 113°$ $AB^2 = (5.2)^2 + (3.58)^2 - 2(5.2)(3.58)\cos 113°$ $AB^2 = 54{,}40410138\ m^2$ $AB = 7{,}38$ m (7,37591.....) Note: AB = 7,3 m or 7,4 m: accept	✓ use of cos rule ✓ substitution ✓ $AB^2 = 54{,}4041...m^2$ ✓ answer (4) Do not penalise if units are omitted.
7.3	*Area of* $\triangle ABL = \frac{1}{2}AL.BL.\sin A\hat{L}B$ $= \frac{1}{2}(5.2)(3.58)\sin 113°$ $= 8.568059176$ $= 8{,}57$ m Note: Area = 8,5 or 8,6 : accept	✓ formula ✓ substitution ✓✓answer (4) If cos $A\hat{L}B$: 0/4 **[10]**

QUESTION 8

8.1	$\cos 3x = \sin x$ $\sin(90° - 3x) = \sin x$ $90° - 3x = x + k.360°$ or $90° - 3x = 180° - x + k.360°$ $k \in Z$ $-4x = -90° + k.360°$ or $-2x = 90° + k.360°$ $x = 22{,}5° - k.90°$ $k \in Z$ $x = -45° - k.180°$ $k \in Z$ $x = -67{,}5°; 22{,}5°; 112{,}5°$ $x = -45°; 135°$	✓ equating ✓ $90° - 3x = x + k.360°$ ✓ $x = 22{,}5° - k.90°$ ✓ $90° - 3x = 180° - x + k.360°$ ✓ $x = -45° - k.180°$ ✓✓✓ values of x (8)
	OR	
	$\cos 3x = \cos(90° - x)$ $3x = 90° - x + k.360°$ or $3x = 360° - (90° - x) + k.360°$ $4x = 90° + k.360°$ or $2x = 270° + k.360°$ $x = 22{,}5° + k.90°$ $k \in Z$ $x = 135° + k.180°$ $k \in Z$ $x = -67{,}5°; 22{,}5°; 112{,}5°$ $x = -45°; 135°$	✓ equating ✓ $3x = 90° - x + k.360°$ ✓ $x = 22{,}5° + k.90°$ ✓ $3x = 360° - (90° - x) + k.360°$ ✓ $x = 135° + k.180°$ ✓✓✓ values of x (8)
	OR	
	$\cos 3x = \cos(90° - x)$ $3x = 90° - x + k.360°$ $3x = -90° - x + k.360°$ $4x = 90° + k.360°$ or $2x = -90° + k.360°$ $x = 22{,}5° + k.90°$ $k \in Z$ $x = -45° - k.180°$ $k \in Z$ $x = -67{,}5°; 22{,}5°; 112{,}5°$ $x = -45°; 135°$	✓ equating ✓ $3x = 90° - x + k.360°$ ✓ $x = 22{,}5° - k.90°$ ✓ $3x = -90° + x + k.360°$ ✓ $x = -45° - k.180°$ ✓✓✓ values of x (8) Note: If not all 5 values for x is given, the following applies 4 or 3 values : 2 marks 2 values : 1 mark 1 value : 0 marks

8.2		(6) Penalise with -1 going beyond the domain.
8.3	$-67{,}5° \le x \le -45°$ **OR** $x \in [-67{,}5°; -45°]$ **OR** From $-67{,}5°$ up to and including $-45°$	✓✓ critical values ✓ notation (3) Note: If $-67{,}5° < x < -45°$: 2/3 Half of the inequality: 1/3 If $x = -67{,}5°$ or $x = -45°$: 0/3 If answer is $22{,}5° \le x \le 112{,}5°$ then 2 / 3 If answer is $135° \le x \le 180°$ then 2 / 3 **[17]**

QUESTION 9

9.1 Mean = $\frac{220}{10}$ = 22 minutes

✓ $\frac{\text{sum of minutes}}{\text{number of runners}}$
✓answer
(2)

9.2

Time taken	$(x-\bar{x})$	$(x_i-\bar{x})^2$
18	-4	16
21	-1	1
16	-6	36
24	2	4
28	6	36
20	-2	4
22	0	0
29	7	49
19	-3	9
23	1	1
Sum		156

Answer only: 2/2

✓✓setting up of table and correct values in column of $(x_i-\bar{x})^2$

$\sigma = \sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n}} = \sqrt{\frac{156}{10}} = 3{,}95$

✓ substitution in formula
✓answer
(4)

(If the candidate used a calculator to answer QUESTION 9.1 and QUESTION 9.2, award full marks if answers are correct.)

If only one mistake in the calculation: 3 / 4

Answer only: 4 / 4

If candidate uses $n-1$ in the formula, the answer

9.3 One standard deviation of the mean is in the interval (22 - 3,95 ; 22 + 3,95) which is (18,05 ; 25,95)

∴ 6 runners completed the race within one standard deviation of the mean.
(List of times: 21, 24, 20, 22, 19, 23)

✓✓ answer
(2)
Answer only : 2 / 2
[8]

If candidate used $\sigma = 4{,}16$, then the interval is (17,84 ; 26,16) and the answer is 7 runners.

QUESTION 10

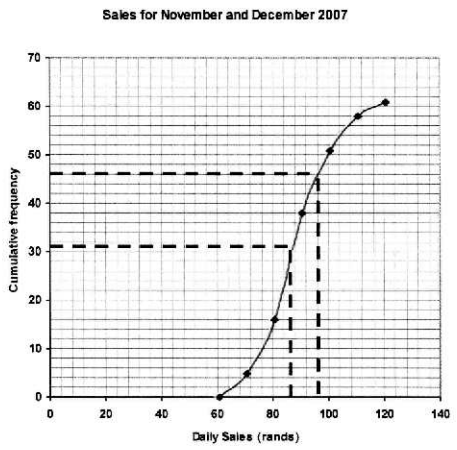
10.1

Daily Sales (in Rand)	Frequency	Cumulative Frequency
60 ≤ rand < 70	5	5
70 ≤ rand < 80	11	16
80 ≤ rand < 90	22	38
90 ≤ rand < 100	13	51
100 ≤ rand < 110	7	58
110 ≤ rand < 120	3	61

✓ Frequency Column
✓✓ cumulative frequencies
(3)

If one wrong in the frequency column, deduct 1 mark.

10.2



✓cumulative totals
✓points at upper limits of intervals
✓shape
(3)

If the ogive is NOT grounded, no penalty.

If plotted as the midpoint of the interval and the cumulative frequency: 2 / 3

10.3 Median = R 87
(Accept answers between 84 and 90)

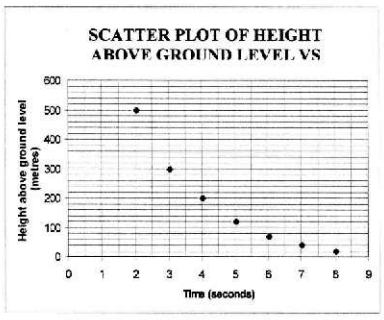
✓correctly read off ogive
(1)

10.4 R 96 ≤ sales ≤ R 120

✓✓correctly read off ogive
(2)
[9]

QUESTION 11

11.1



✓✓ all points plotted correctly.
(2)

No penalty if the points are joined.

11.2	Exponential OR Quadratic OR Hyperbola OR Decreasing steeply then gradually. (Applicable descriptions are acceptable)	✓ answer (1) Straight line : 0 / 1
11.3	Approximately 90 m	✓ answer (1) **[4]**

QUESTION 12

12.1	The median, the maximum scores, IQR Note: Any two statements that are valid in the context of the problem apply.	✓✓ any two of the list (2)
12.2	IQR = 90 – 72 = 18.	✓ formula ✓ answer (2) Answer only: 2 / 2
12.3	No. In the calculation of the median only the value in the middle of an ordered data set is of importance. The extreme values are not taken into account. In this case, 25% of the learners in Class A had a score of less than 66 marks. The minimum mark in Class B is 66 marks. Hence the performance of the two classes differ significantly. OR No. The one is skewed to the left and the other is skewed to the right. The extreme values are not taken into account. OR No. The lower quartile of Class A is below the minimum of Class B. The extreme values are not taken into account. OR No. The left whisker of Class A is much longer than the left whisker of Class B. The extreme values are not taken into account.	✓ No ✓ extreme values not taken into account ✓ minimum marks different (3) **[7]**

TOTAL: 150 marks

education
DEPARTMENT: EDUCATION
MPUMALANGA PROVINCE

Grade 12 Education Supplement 2010

Mathematics

Mathematic Paper 01
November 2010

Mathematic Paper 01
November 2008

Together Educating the Nation

QUESTION 1

1.1 Solve for x, rounded off to TWO decimal places where necessary:

1.1.1 $x^2 = 5x - 4$ (3)

1.1.2 $x(3 - x) = -3$ (5)

1.1.3 $3 - x < 2x^2$ (5)

1.2 Determine the values of x and y if they satisfy both the following equations simultaneously:

$2x + y = 3$
$x^2 + y + x = y^2$ (8)

1.3 Given $x = 999\,999\,999\,999$, determine the exact value of $\frac{x^2 - 4}{x - 2}$.
Show ALL your calculations. (3)

1.4 Explain why the equation $\frac{x^4 + 1}{x^4} = \frac{1}{2}$ has no real roots. (2)
[26]

QUESTION 2

2.1 Consider the sequence: $\frac{1}{2}; 4; \frac{1}{4}; 7; \frac{1}{8}; 10; \ldots$

2.1.1 If the pattern continues in the same way, write down the next TWO terms in the sequence. (2)

2.1.2 Calculate the sum of the first 50 terms of the sequence. (7)

2.2 Consider the sequence: 8 ; 18 ; 30 ; 44 ; …

2.2.1 Write down the next TWO terms of the sequence, if the pattern continues in the same way. (2)

2.2.2 Calculate the n^{th} term of the sequence. (6)

2.2.3 Which term of the sequence is 330? (4)
[21]

QUESTION 3

Given the geometric series: $8x^2 + 4x^3 + 2x^4 + \ldots$

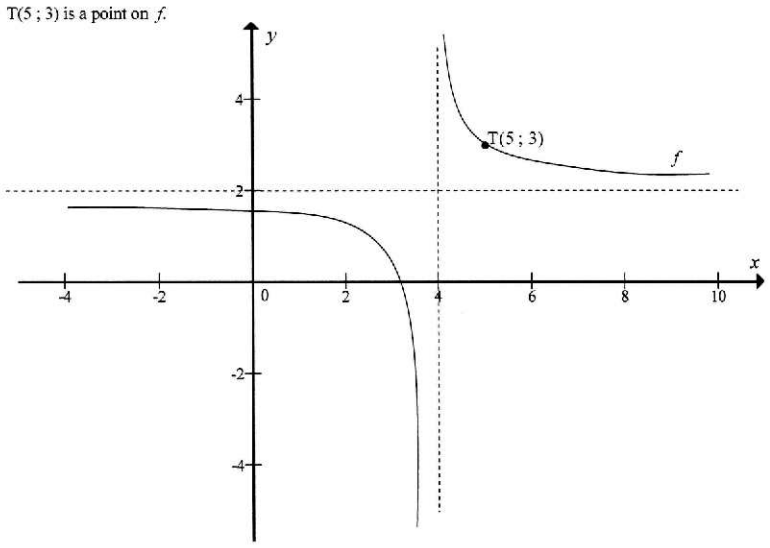
3.1 Determine the n^{th} term of the series. (1)

3.2 For what value(s) of x will the series converge? (3)

3.3 Calculate the sum of the series to infinity if $x = \frac{3}{2}$. (3)
[7]

QUESTION 4

The diagram below represents the graph of $f(x) = \frac{a}{x - p} + q$.
T(5 ; 3) is a point on f.



4.1 Determine the values of a, p and q. (4)

4.2 If the graph of f is reflected across the line having equation $y = -x + c$, the new graph coincides with the graph of $y = f(x)$. Determine the value of c. (3)
[7]

QUESTION 5

Given: $h(x) = 4^x$ and $f(x) = 2(x - 1)^2 - 8$.

5.1 Sketch the graphs of h and f on the diagram sheet provided. Indicate ALL intercepts with the axes and any turning points. (8)

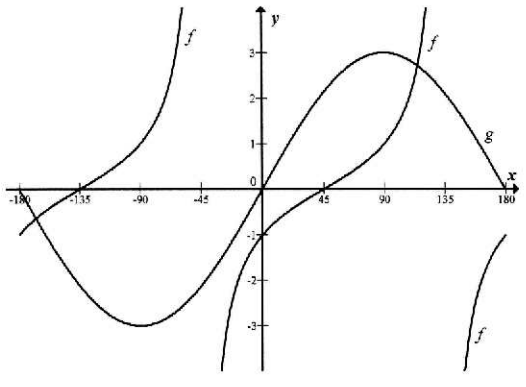
5.2 Without any further calculations, sketch the graph of $y = \log_4 x = g(x)$ on the same system of axes. (2)

5.3 The graph of f is shifted 2 units to the LEFT. Write down the equation of the new graph. (2)

5.4 Show, algebraically, that $h\left(x + \frac{1}{2}\right) = 2h(x)$. (3)
[15]

QUESTION 6

Sketched below are the graphs of the functions $f(x) = \tan(x - 45°)$ and $g(x) = 3\sin x$ for $x \in [-180° ; 180°]$.



6.1 Write down the equations of the asymptotes of $y = f(x)$ for $x \in [-90° ; 180°]$. (2)

6.2 Describe the transformation of the graph of f to h if $h(x) = \tan(45° - x)$. (2)

6.3 The period of g is reduced to 180° and the amplitude and y-intercept remain the same. Write down the equation of the resulting function. (2)
[6]

QUESTION 7

7.1 R1 570 is invested at 12% p.a. compound interest. After how many years will the investment be worth R23 000? (4)

7.2 A farmer has just bought a new tractor for R800 000. He has decided to replace the tractor in 5 years' time, when its trade-in value will be R200 000. The replacement cost of the tractor is expected to increase by 8% per annum.

7.2.1 The farmer wants to replace his present tractor with a new one in 5 years' time. The farmer wants to pay cash for the new tractor, after trading in his present tractor for R200 000. How much will he need to pay? (3)

7.2.2
- One month after purchasing his present tractor, the farmer deposited x rands into an account that pays interest at a rate of 12% p.a., compounded monthly.
- He continued to deposit the same amount at the end of each month for a total of 60 months.
- At the end of 60 months he has exactly the amount that is needed to purchase a new tractor, after he trades in his present tractor.

Calculate the value of x. (6)

7.2.3 Suppose that 12 months after the purchase of the present tractor and every 12 months thereafter, he withdraws R5 000 from his account, to pay for maintenance of the tractor. If he makes 5 such withdrawals, what will the new monthly deposit be? (4)
[17]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = -3x^2$. (5)

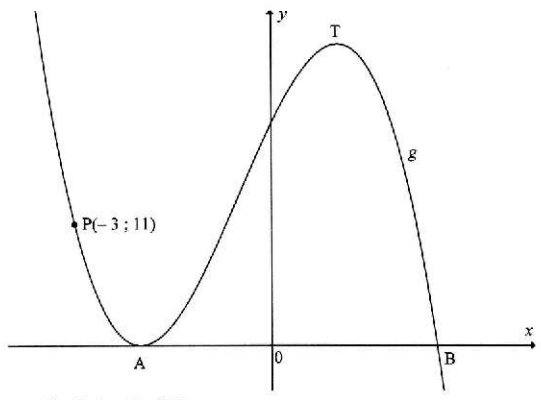
8.2 Determine, using the rules of differentiation:

$$\frac{dy}{dx} \text{ if } y = \frac{\sqrt{x}}{2} - \frac{1}{6x^3}$$

Show ALL calculations. (3)
[8]

QUESTION 9

Sketched below is the graph of $g(x) = -2x^3 - 3x^2 + 12x + 20 = -(2x - 5)(x + 2)^2$
A and T are turning points of g. A and B are the x-intercepts of g.
P(– 3 ; 11) is a point on the graph.



9.1 Determine the length of AB. (2)

9.2 Determine the x-coordinate of T. (4)

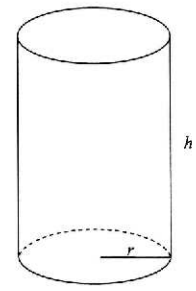
9.3 Determine the equation of the tangent to g at P(– 3 ; 11), in the form $y = \ldots$ (5)

9.4 Determine the value(s) of k for which $-2x^3 - 3x^2 + 12x + 20 = k$ has three distinct roots. (3)

9.5 Determine the x-coordinate of the point of inflection. (4)
[18]

QUESTION 10

A drinking glass, in the shape of a cylinder, must hold 200 mℓ of liquid when full.



10.1 Show that the height of the glass, h, can be expressed as $h = \frac{200}{\pi r^2}$. (2)

10.2 Show that the total surface area of the glass can be expressed as $S(r) = \pi r^2 + \frac{400}{r}$. (2)

10.3 Hence determine the value of r for which the total surface area of the glass is a minimum. (5)
[9]

QUESTION 11

Amina owns a small factory that manufactures two types of cellular phones, namely Acuna and Matata cellular phones.

- Each Acuna cellular phone requires 10 manhours to manufacture and each Matata cellular phone requires 8 manhours to manufacture.
- Each Acuna cellular phone requires 3 manhours in the testing department and each Matata cellular phone requires 4 manhours in the testing department.
- The manufacturing department has a maximum of 800 manhours available per week.
- The testing department has a maximum of 360 manhours available per week.
- The factory needs to manufacture at least 60 of the Matata models each week.

Let x represent the number of Acuna cellular phones manufactured in one week.
Let y represent the number of Matata cellular phones manufactured in one week.

11.1 Write down the constraints, in terms of x and y, that represent the above-mentioned information. (3)

11.2 Use the attached graph paper (DIAGRAM SHEET 2) to represent the constraints graphically. (5)

11.3 Clearly indicate the feasible region by shading it. (1)

11.4 If the profit on one Acuna cellular phone is R200 and the profit on one Matata cellular phone is R250, write down an expression that will represent the profit, P, on the cellular phones. (1)

11.5 Using a search line and your graph, determine the number of Acuna and Matata cellular phones that will give a maximum profit, assuming they are all sold out. Draw a search line on your graph. (3)

11.6 If the profit function for the factory was P = 180x + 240y, would there be any difference in the optimal solution? Give a reason for your answer. (3)
[16]

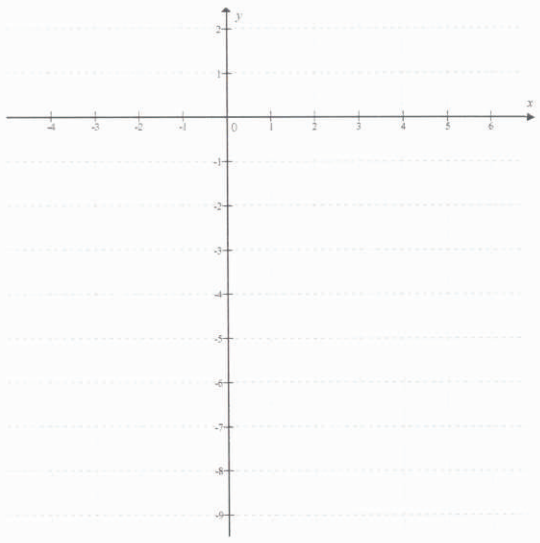
TOTAL: 150

Mathematics/P1 NSC DoE/November 2008

EXAMINATION NUMBER:

DIAGRAM SHEET 1

QUESTIONS 5.1 AND 5.2

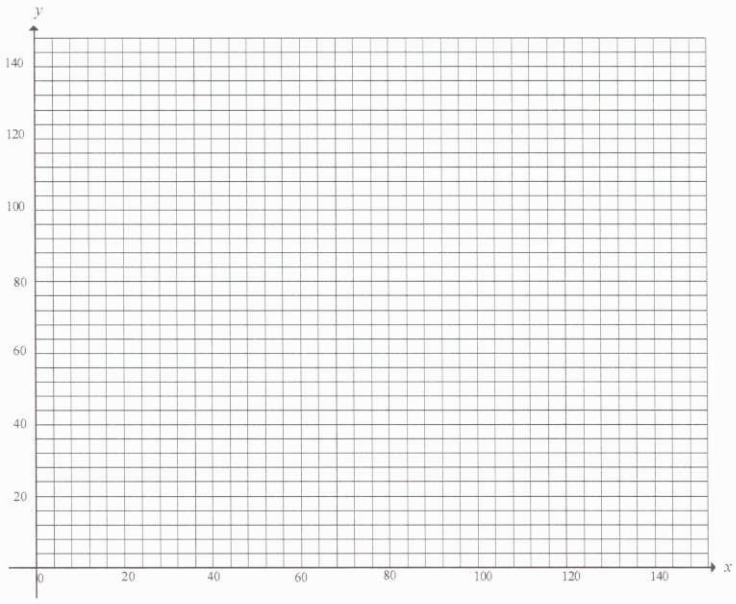


Mathematics/P1 NSC DoE/November 2008

EXAMINATION NUMBER:

DIAGRAM SHEET 2

QUESTIONS 11.2 AND 11.3



Mathematics/P1 NSC DoE/November 2008

INFORMATION SHEET: MATHEMATICS
INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} \left(a + (i-1)d\right) = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$\sum_{i=1}^{n} ar^{i-1} = \frac{a\left(r^n - 1\right)}{r-1} \;;\quad r \neq 1 \qquad \sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1-r} \;;\; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A \qquad \text{area } \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

- **Continued Accuracy will apply as a general rule.**
- **If a candidate does a question twice and does not delete either, only mark the FIRST attempt.**
- **If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.**

QUESTION 1

1.1.1

$x^2 = 5x - 4$
$x^2 - 5x + 4 = 0$
$(x-4)(x-1) = 0$
$x = 4$ or $x = 1$

– 1 for not equal to zero in this question only. If = 0 appears once in this question then full marks

✓ standard form = 0
✓ factorisation
✓ both answers
(3)

OR

By the formula
✓ standard form = 0
✓ substitution
✓ both answers

1.1.2

$x(3-x) = -3$
$3x - x^2 = -3$
$x^2 - 3x - 3 = 0$
$x = \dfrac{3 \pm \sqrt{(-3)^2 - 4(1)(-3)}}{2(1)}$
$x = \dfrac{3 \pm \sqrt{21}}{2}$
$x = 3{,}79$ or $x = -0{,}79$

– 1 for inaccurate rounding off for both answers.

✓ simplification
✓ standard form
✓ substitution into formula
✓✓ answers
(5)

OR

$x(3-x) = -3$
$3x - x^2 = -3$
$-x^2 + 3x + 3 = 0$
$x = \dfrac{-3 \pm \sqrt{(3)^2 - 4(-1)(3)}}{2(-1)}$
$x = \dfrac{-3 \pm \sqrt{21}}{-2}$
$x = 3{,}79$ or $x = -0{,}79$

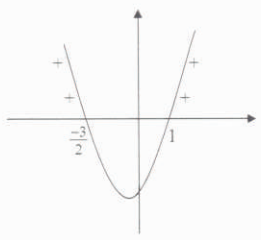
– 1 for inaccurate rounding off for both answers.

✓ simplification
✓ standard form
✓ substitution into formula
✓✓ answers
(5)

Note:
If negative discriminant: max 2 / 5

1.1.3

$3 - x < 2x^2$
$-2x^2 - x + 3 < 0$
$2x^2 + x - 3 > 0$
$(2x+3)(x-1) > 0$
$x < -\dfrac{3}{2}$ or $x > 1$
OR
$x \in (-\infty; -\frac{3}{2}) \cup (1; \infty)$



✓ standard form
✓ factorisation
✓ OR / ∪
✓ $x < -\frac{3}{2}$
✓ $x > 1$
(5)

OR

$3 - x < 2x^2$
$0 < 2x^2 + x - 3$
$0 < (2x+3)(x-1)$
$x < -\dfrac{3}{2}$ or $x > 1$

✓ standard form
✓ factorisation
✓ OR / ∪
✓ $x < -\frac{3}{2}$
✓ $x > 1$
(5)

Note:
4 / 5 Inaccurate inequality in the beginning
2 / 5 If final answer does not have inequality signs (ie. question has been changed to an equation)
4 / 5 If the candidate has used AND or ∩ instead of OR or ∪

If Answer is
$(2x+3)(x-1) > 0$
$-\dfrac{3}{2} < x < 1$ then: 2 / 5

1.2

$y = 3 - 2x$
$x^2 + (3-2x) + x = (3-2x)^2$
$x^2 + 3 - 2x + x = 9 - 12x + 4x^2$
$3x^2 - 11x + 6 = 0$
$(3x-2)(x-3) = 0$
$x = \dfrac{2}{3}$ or $x = 3$
$\therefore y = \dfrac{5}{3}$ $\therefore y = -3$

✓ $y = 3 - 2x$
✓ substitution
✓ simplification of $(3-2x)^2$
✓ standard form
✓ factorisation
✓ both x values
✓✓ y values
(8)

OR

$x = \dfrac{3-y}{2}$
$\left(\dfrac{3-y}{2}\right)^2 + y + \dfrac{3-y}{2} = y^2$
$\dfrac{9-6y+y^2}{4} + y + \dfrac{3-y}{2} = y^2$
$9 - 6y + y^2 + 4y + 6 - 2y = 4y^2$
$0 = 3y^2 + 4y - 15$
$0 = (3y-5)(y+3)$
$y = \dfrac{5}{3}$ or $y = -3$
$\therefore x = \dfrac{2}{3}$ $\therefore x = 3$

✓ $x = \dfrac{3-y}{2}$
✓ substitution
✓ simplification of $\left(\dfrac{3-y}{2}\right)^2$
✓ standard form
✓ factorisation
✓ both y values
✓✓ x values
(8)

OR

$y = 3 - 2x$
$x^2 - y^2 + x + y = 0$
$(x+y)(x-y) + (x+y) = 0$
$(x+y)(x-y+1) = 0$

$y = -x$ $\quad$ $y = x + 1$
$3 - 2x = -x$ $\quad$ or $\quad$ $3 - 2x = x + 1$
$x = 3$ $\quad$ $x = \dfrac{2}{3}$
$y = -3$ $\quad$ $y = \dfrac{5}{3}$

✓ $y = 3 - 2x$
✓ common factor
✓ common bracket
✓ $y = -x$
✓ $3 - 2x = -x$
✓ both x-values
✓✓ y-values
(8)

OR

$x = \dfrac{3-y}{2}$
$x^2 - y^2 + x + y = 0$
$(x+y)(x-y) + (x+y) = 0$
$(x+y)(x-y+1) = 0$

$y = -x$ $\quad$ $y = x + 1$
$y = -\dfrac{3-y}{2}$ $\quad$ $y = \dfrac{3-y}{2} + 1$
$2y = -3 + y$ $\quad$ or $\quad$ $2y = 3 - y + 2$
$y = -3$ $\quad$ $3y = 5$
$x = 3$ $\quad$ $y = \dfrac{5}{3}$
$x = \dfrac{2}{3}$

✓ $x = \dfrac{3-y}{2}$
✓ common factor
✓ common bracket
✓ $y = -x$
✓ $y = -\dfrac{3-y}{2}$
✓ both y-values
✓✓ x-values
(8)

1.3

$\dfrac{x^2-4}{x-2} = \dfrac{(x+2)(x-2)}{(x-2)} = x + 2$

Therefore when $x = 999\,999\,999\,999$, the value is $999\,999\,999\,999 + 2 = 1\,000\,000\,000\,001$.

OR

$\dfrac{x^2-4}{x-2} = \dfrac{(x+2)(x-2)}{(x-2)} = x + 2$
$999\,999\,999\,999 = 10^{12} - 1$
$x + 2 = 999\,999\,999\,999 + 2$
$= 10^{12} + 1$

✓ factorisation
✓ simplification
✓ answer
(3)

Note:
If candidate has substituted directly, 0/3 (answer would be 1×10^{12} by substitution)
Answer only: 2 / 3
Correct answer but incorrect mathematics 0 / 3

1.4

$\dfrac{x^4+1}{x^4} = 1 + \dfrac{1}{x^4} > 1$ since $\dfrac{1}{x^4} > 0$
$\therefore \dfrac{x^4+1}{x^4}$ can never be equal to ½

✓ inequality
✓ conclusion
(2)

OR

$2x^4 + 2 = x^4$
$\dfrac{1}{x^4} = -\dfrac{1}{2}$
Which has no real solution since $\dfrac{1}{x^4} > 0$ for all $x \in R - \{0\}$

✓ equation
✓ conclusion
(2)

OR

	$2x^4 + 2 = x^4$ $x^4 + 2 = 0$ $x^4 + 0x^2 + 2 = 0$ $b^2 - 4ac = 0 - 4(1)(2)$ $= -8$ < 0 $\therefore$ no real roots	✓ calculation ✓ $\Delta < 0$ or $\Delta = -8$ (2)
	$2x^4 + 2 = x^4$ $\therefore x^4 = -2$ Which has no real solution since $x^4 \geq 0$ for all $x \in R$	✓ equation ✓ conclusion (2) **[26]**

QUESTION 2

2.1.1	$\frac{1}{16}$; 13	✓✓ answers (2)
2.1.2	$\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+...\text{to 25 terms}\right)$ $(4+7+10+13+...\text{to 25 terms})$ $\frac{a(r^n-1)}{r-1}$ $= \frac{n}{2}[2a+(n-1)d]$ $= \frac{\frac{1}{2}\left(\left(\frac{1}{2}\right)^{25}-1\right)}{\frac{1}{2}-1}$ $= \frac{25}{2}[2(4)+24(3)]$ $= 0{,}9999999$ $= 1\,000$ $S_{50} = 1001{,}00$ **OR** S_{50} = 25 terms of 1st sequence + 25 terms of 2nd sequence $S_{50} = \left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+...\text{to 25 terms}\right)+(4+7+10+13+...\text{to 25 terms})$ $S_{50} = \frac{\frac{1}{2}\left(\left(\frac{1}{2}\right)^{25}-1\right)}{\frac{1}{2}-1}+\frac{25}{2}[2(4)+24(3)]$ $S_{50} = 0{,}999999...+1000$ $S_{50} = 1001{,}00$	✓ formula for geometric series ✓ $\frac{\frac{1}{2}\left(\left(\frac{1}{2}\right)^{25}-1\right)}{\frac{1}{2}-1}$ ✓ answer for geometric series ✓ formula for linear series ✓ $\frac{25}{2}[2(4)+24(3)]$ ✓ 1000 ✓ answer (7) Note: If used 50 terms in each series: max 5/7 (answer then is 3876) Answer only: 6 / 7 Write out series and then correct answer: full marks Write out both series and not add them: 6 / 7

	T_0 T_1 T_2 T_4 T_5 0 8 18 30 44 8 10 12 14 2 2 2 $T_0 = 0$ $a(0)^2 + b(0) + c = 0$ $c = 0$ constant second difference = 2 $a = 1$ $T_1 = 1 + b = 8$ $b = 7$ $T_n = n^2 + 7n$ $T_n = n(n+7)$ **OR** $T_n = \frac{n-1}{2}[2(\text{first first difference}) + (n-2)(\text{second difference})] + T_1$ $T_n = \frac{n-1}{2}[2(10) + (n-2)(2)] + 8$ $T_n = 10(n-1) + (n-2)(n-1) + 8$ $T_n = 10n - 10 + n^2 - 3n + 2 + 8$ $T_n = n^2 + 7n$ **OR** $T_n = (n-1)T_2 - (n-2)T_1 + 2\text{nd difference}\frac{(n-1)(n-2)}{2}$ $T_n = (n-1)(18) - (n-2)(8) + 2\frac{(n-1)(n-2)}{2}$ $T_n = 18n - 18 - 8n + 16 + n^2 - 3n + 2$ $T_n = n^2 + 7n$ **OR**	✓ finding T_0 ✓ $c = 0$ ✓ second difference = 2 ✓ $a = 1$ ✓ substitution ✓ $b = 7$ (6) ✓ formula ✓✓ substitution ✓✓ simplification ✓ answer (6) ✓✓ formula ✓✓ substitution ✓ simplification ✓ answer (6)

2.2.1	60 ; 78	✓✓ answers (2)
2.2.2	8 18 30 44 10 12 14 2 2 $2a = 2$ $a = 1$ $T_n = n^2 + bn + c$ $8 = 1 + b + c$ $7 = b + c$ ...(i) $18 = 4 + 2b + c$ $14 = 2b + c$ ...(ii) (ii) – (i): $14 = 2b + c$ $7 = b + c$ $\therefore 7 = b$ $c = 0$ $T_n = n^2 + 7n$ **OR**	✓ $a = 1$ ✓ substitution ✓ solving simultaneously ✓ $b = 7$ ✓ $c = 0$ ✓ general term (6)
	$T_1 = 8$ $T_2 - T_1 = 10$ $T_3 - T_2 = 12$ $T_n - T_{n-1} =$ nth term of sequence with $a = 8$ and $d = 2$ Add both sides $T_n = 8 + 10 + 12 + ...$ + to 25 terms $T_n = \frac{n}{2}[16 + 2(n-1)]$ $T_n = n(n+7)$ **OR**	✓ $T_1 = 8$ ✓ $T_2 - T_1 = 10$ ✓ $T_3 - T_2 = 12$ ✓ Add both sides ✓ sequence ✓ substitution (6)

	$T_n = \frac{(n-2)(n-3)T_1 - 2(n-1)(n-3)T_2 + (n-2)(n-1)T_3}{2}$ $T_n = \frac{(n^2-5n+6)(8) - 2(n^2-4n+3)(18) + (n^2-3n+2)(30)}{2}$ $T_n = 4n^2 - 20n + 24 - 18n^2 + 72n - 54 + 15n^2 - 45n + 30$ $T_n = n^2 + 7n$ **OR** $T_1 = 8 = 1.8$ $T_2 = 18 = 2.9$ $T_3 = 30 = 3.10$ $T_4 = 44 = 4.11$ $T_n = n^2 + 7n$		✓ formula ✓✓ substitution ✓✓ simplification ✓ answer (6) ✓✓✓✓✓ observation ✓ answer (6) Note: By trial and error: 6 / 6 Answer only: 6 / 6
2.2.3	$n(n+7) = 330$ $n^2 + 7n - 330 = 0$ $(n+22)(n-15) = 0$ $n = -22$ or $n = 15$ $n = 15$ $\therefore 15^{th}$ term is 330.	Note: 3 / 4 if did not reject $n = -22$ Answer only: 4 / 4 By trial and error and then write n = 15: 4 / 4 1 / 4 if just equate T_n that they found If linear T_n and valid answer : 2 / 4	✓ substitution ✓ standard form ✓ factorisation ✓ answer (4) **[21]**

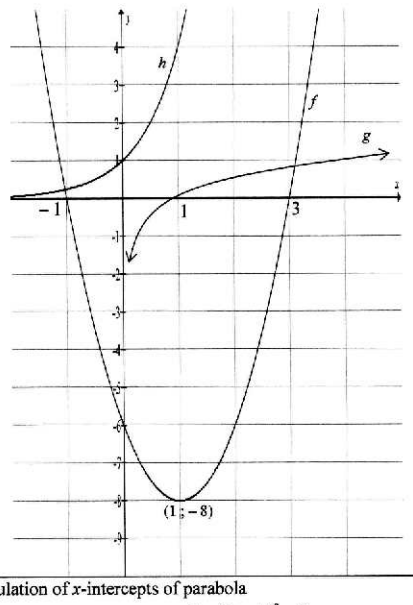
QUESTION 3

3.1	$T_n = (8x^2)\left(\frac{x}{2}\right)^{n-1}$ **OR** $T_n = 8\left(\frac{1}{2}\right)^{n-1}.x^{n+1}$ **OR** $T_n = 16x\left(\frac{x}{2}\right)^n$ **OR** $T_n = 2^{4-n}x^{n+1}$	✓ answer (1)
3.2	$ratio = \frac{x}{2}$ $-1 < \frac{x}{2} < 1$ $-2 < x < 2$	✓ ratio ✓ inequality ✓ answer (3)
3.3	$S_\infty = \frac{a}{1-r}$ $S_\infty = \frac{8x^2}{1-\frac{x}{2}}$ $S_\infty = \frac{8\left(\frac{3}{2}\right)^2}{1-\frac{1}{2}\left(\frac{3}{2}\right)}$ $S_\infty = 72$ **OR** $18+\frac{27}{2}+\frac{81}{8}+\ldots$ $S_\infty = \frac{18}{1-\frac{3}{4}}$ $S_\infty = \frac{18}{\frac{1}{4}}$ $S_\infty = 72$	✓ substitution into formula for S_∞ ✓ substitution of $x = \frac{3}{2}$ ✓ answer (3) ✓ series ✓ substitution ✓ answer (3) Formula Incorrect: 0 / 3 [7]

QUESTION 4

4.1	$p = 4$ $q = 2$ $3 = \frac{a}{5-4}+2$ $1 = \frac{a}{1}$ $a = 1$	✓ answer p ✓ answer q ✓ substitution of (5; 3) ✓ answer (4) Answer for p 1 mark Answer for q 1 mark Answer for a 2 marks
4.2	$y = -x + c$ substitute (4 ; 2) $2 = -4 + c$ $c = 6$ **OR** Translation of the line $y = -x$ 2 units up and 4 units right $y = -(x-4)+2$ $y = -x+6$	✓ correct point (4 ; 2) ✓ substitution ✓ answer (3) ✓ substitution of $x-4$ ✓ adding 2 ✓ answer (3) Substitution of T(3 ; 5): 0 / 3 Answer only: 3 / 3 [7]

QUESTION 5

5.1 & 5.2		EXPONENTIAL ✓ shape (must be increasing above x-axis) ✓ y-int PARABOLA ✓ shape ✓✓ turning point ✓ y-intercept ✓✓ x-intercepts (8) INVERSE/LOG ✓ x-int ✓ shape (must be increasing on the right of the y-axis) (2) Note: If x-intercepts not shown but correct on graph 2/2 for x-intercepts.
	Calculation of x-intercepts of parabola $0 = 2(x-1)^2 - 8$ $8 = 2(x-1)^2$ $4 = (x-1)^2$ $2 = x-1$ *or* $-2 = x-1$ $x = 3$ *or* $x = -1$ OR $0 = 2(x-1)^2 - 8$ $0 = 2(x^2-2x+1)-8$ $0 = 2x^2 - 4x - 6$ $0 = x^2 - 2x - 3$ $0 = (x-3)(x+1)$ $x = 3$ *or* $x = -1$	
5.3	$y = 2(x+1)^2 - 8$ **OR** $y = 2x^2 + 4x - 6$	✓ −8 ✓ +1 (2) ✓ −6 ✓ +4 (2)

5.4	$h\left(x+\frac{1}{2}\right) = 4^{x+\frac{1}{2}}$ $= 4^x.4^{\frac{1}{2}}$ $= 2(4^x)$ $= 2h(x)$ **OR** $h\left(x+\frac{1}{2}\right) = 4^{x+\frac{1}{2}}$ $= (2^2)^{x-\frac{1}{2}}$ $= 2^{2x-1}$ $= 2^{2x}.2$ $= 2.(4^x)$ $= 2h(x)$	✓ substitution ✓ $4^x.4^{\frac{1}{2}}$ ✓ $2(4^x)$ (3) ✓ substitution ✓ $(2^2)^{x-\frac{1}{2}}$ ✓ $2(4^x)$ (3) Note: If numerical examples are used : 1 / 3 [15]

QUESTION 6

6.1	$x = -45°$ $x = 135°$	✓ answer ✓ answer (2) Note: If correct numbers but not writing as an equation 1 / 2 If units left out: 2 / 2
6.2	$h(x) = \tan(45° - x)$ $h(x) = -\tan(x - 45°) = -f(x)$ h is the reflection of f about the x-axis **OR** h is the reflection of f about the line $y = 0$	✓✓ reflection about x-axis (2) ✓✓ reflection about $y = 0$ (2) Note: If calculation only: 1 /2 If answer is: Reflection only: 0 / 2 If do calculation and say reflection: 1 / 2 Only $h(x) = \tan(45° - x)$ $h(x) = -\tan(x - 45°) = -f(x)$ 1 / 2
6.3	$y = 3\sin 2x$	✓ 3 ✓ $2x$ (2) **[6]**

QUESTION 7

Penalise ONCE in question 7 for early rounding off.

7.1	$A = P(1+i)^n$ $23000 = 1570(1.12)^n$ $(1.12)^n = 14{,}64968153..$ $n\log(1{,}12) = \log 14{,}64968153..$ $n = 23{,}69$ years (23,68701...) or $n = 24$ years or $n = 23$ years 8 months or $n = 23{,}7$ years **OR** $A = P(1+i)^n$ $23000 = 1570(1 + \frac{12}{100})^n$ $(1.12)^n = 14{,}64968153..$ $n\log(1{,}12) = \log 14{,}64968153..$ $n = 23{,}69$ years (23,68701...) or $n = 24$ years or $n = 23$ years 8 months or $n = 23{,}7$ years Note: Accept 24 years : 4 / 4 Incorrect Formula: 0/4	✓ formula ✓ substitution ✓ apply log function ✓answer (4) ✓ formula ✓ substitution of $\frac{12}{100}$ ✓ apply log function ✓answer (4)
7.2.1	$A = P(1+i)^n$ $= 800000(1.08)^5$ $= R1175462{,}46$ $\therefore R1175462{,}46 - R200\,000$ $= R975462{,}46$ Some calculators give R 975 462,50	✓ substitution ✓ R 1 175 462,46 ✓ R 975 462,46 (3) Incorrect Formula: 0/3
7.2.2	$F = \frac{x[(1+i)^n - 1]}{i}$ $975462{,}46 = x\frac{[1{,}01]^{60} - 1}{0{,}01}$ $\frac{975462{,}46 \times 0.01}{[1{,}01]^{60} - 1} = x$ $x = R11944{,}00$	✓ F = R975462,46 or answer in 7.2.1 ✓ $n = 60$ ✓ $i = 1{,}01$ ✓ formula ✓ simplification ✓ answer (6)

	OR $975462{,}46 = x\frac{[1{,}01]^{60} - 1}{0{,}01}$ $975462{,}46 = 81{,}66966986x$ $x = R11944{,}00$	✓ F = R975462,46 ✓ $n = 60$ ✓ $i = 1{,}01$ ✓ formula ✓ simplification ✓ answer (6) Note: Continued Accuracy applies.

7.2.3	$Service = [5000(1{,}01)^{48} + 5000(1{,}01)^{36} + 5000(1{,}01)^{24} + 5000(1{,}01)^{12} + 5000]$ $= 32197{,}77$ $975462{,}46 = x\frac{[1{,}01]^{60} - 1}{0{,}01} - Service$ $975462{,}46 = 81{,}66966986x - 32197{,}77$ $x = R12338{,}24$ **OR** $Service = \frac{5000[1{,}01^{60} - 1]}{1{,}01^{12} - 1}$ $= 32197{,}77$ $975462{,}46 = x\frac{[1{,}01]^{60} - 1}{0{,}01} - Service$ $975462{,}46 = 81{,}66966986x - 32197{,}77$ $x = R12338{,}24$ **OR** Present Value payment of R 5000 $= 5000\{(1{,}01)^{-12} + (1{,}01)^{-24} + (1{,}01)^{-36} + (1{,}01)^{-48} + (1{,}01)^{-60}\}$ $= 5000(1{,}01)^{-12}\left\{\frac{1-(1{,}01)^{-60}}{1-(1{,}01)^{-12}}\right\}$ $= R\ 17\ 723{,}25$ Present Value of the sinking fund $= 975462{,}46(1{,}01)^{-60}$ $= R\ 536\ 942{,}94$ Total Value of sinking fund $= R\ 17\ 723{,}25 + R\ 536\ 942{,}94$ $= R\ 554\ 666{,}19$ $\therefore 554666{,}19 = x\left\{\frac{1-(1{,}01)^{-60}}{0{,}01}\right\}$ $x = R\ 12\ 338{,}24$ **OR**	✓✓ 32 197,77 ✓ setting up of correct equation ✓ answer (4) ✓✓ 32 197,77 ✓ setting up of correct equation ✓ answer (4) ✓ 17723,25 ✓ 554666,19 ✓ setting up of correct equation ✓ answer (4)

	$(1 + i_{eff}) = (1 + 0{,}01)^{12}$ $i_{eff} = 0{,}12682503.....$ $P(1+i)^n$ $= 5000\frac{(1{,}12682503)^5 - 1}{0{,}12682503}$ $= 32197{,}77$ $975462{,}46 = x\frac{[1{,}01]^{60} - 1}{0{,}01} - 32197{,}77$ $975462{,}46 = 81{,}66966986x - 32197{,}77$ $x = R12338{,}24$ **OR** $5000 = \frac{x[(1{,}01)^{12} - 1)}{0{,}01}$ $x = \frac{5000 \times 0{,}01}{1{,}01^{12} - 1}$ $x = 394{,}24$ So monthly deposit must be increased by R 394,24 New monthly deposit = R 11 944 + R 394,24 = R 12 338,24	✓ substitution into formula ✓ 32 197,77 ✓ setting up of correct equation ✓ answer R 12 338,24 (4) ✓ substitution into formula ✓ 394,24 ✓ setting up of correct equation ✓ answer R 12 338,24 (4) **[17]**

QUESTION 8

8.1	$f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to0}\frac{-3(x+h)^2-(-3x^2)}{h}$ $=\lim_{h\to0}\frac{-3x^2-6xh-3h^2+3x^2}{h}$ $=\lim_{h\to0}\frac{-6xh-3h^2}{h}$ $=\lim_{h\to0}\frac{h(-6x-3h)}{h}$ $=\lim_{h\to0}(-6x-3h)$ $=-6x$	✓✓ definition $\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$ ✓ $-3(x+h)^2$ ✓substitution of $-3x^2$ ✓correct answer (5) Note: Penalty 1 for incorrect notation If a candidate has used the rules only: 0/5
8.2	$y=\frac{\sqrt{x}}{2}-\frac{1}{6x^3}$ $y=\frac{1}{2}x^{\frac{1}{2}}-\frac{1}{6}x^{-3}$ $\frac{dy}{dx}=\frac{1}{4}x^{-\frac{1}{2}}+\frac{3}{6}x^{-4}$ $\frac{dy}{dx}=\frac{1}{4}x^{-\frac{1}{2}}+\frac{1}{2}x^{-4}$ $\frac{dy}{dx}=\frac{1}{4\sqrt{x}}+\frac{1}{2x^4}$ Note: If removed coefficients, or moved the numbers from the denominator to the numerator: **Continued accuracy applies for each correct derivative Max 2/3** If leave out $\frac{dy}{dx}$ penalise 1 mark.	✓ Simplification ✓ $\frac{1}{4}x^{-\frac{1}{2}}$ ✓ $\frac{1}{2}x^{-4}$ or $\frac{3}{6}x^{-4}$ (3) **[8]**

9.4	y-coordinate of T is $g(1)=-2(1)^3-3(1)^2+12(1)+20$ $=27$ T(1 ; 27) $\therefore 0<k<27$ **OR** $-2x^3-3x^2+12x+20=k$ $-2x^3-3x^2+12x+20-k=0$ $-7<20-k<20$ $-27<-k<0$ $0<k<27$	✓ y-coordinate of T (27) ✓✓ answer (3) ✓ $-7<20-k<20$ ✓✓ answer (3) Answer Only: 3/3 $0\le k\le 27$: 2 / 3 $k>0$: 1 / 3 $k<27$: 1 / 3
9.5	$g'(x)=-6x^2-6x+12$ $g''(x)=-12x-6$ $12x+6=0$ $x=-\frac{1}{2}$ $g''(x)<0$ $g''(x)>0$ at $x=-\frac{1}{2}$ $g''(x)$ changes sign at $x=-\frac{1}{2}$ $\therefore$ point of inflection at $x=-\frac{1}{2}$ **OR** Turning points A(-2;0); T(1;27) Now x co-ordinate of point of inflection is $x=-\frac{-2+1}{2}=-\frac{1}{2}$	✓ $-12x$ ✓ -6 ✓ $=0$ ✓ $x=-\frac{1}{2}$ (4) ✓✓ points ✓✓ $x=-\frac{1}{2}$ (4) **[18]**

QUESTION 9

9.1	$-(2x-5)(x+2)=0$ $x=\frac{5}{2}or-2$ AB = 4,5 units **OR** $-(2x-5)(x+2)=0$ $x=\frac{5}{2}or-2$ AB $=\sqrt{(2{,}5-(-2)^2+(0-0)^2}$ AB = 4,5 units	✓ $x=\frac{5}{2};x=-2$ ✓ answer (2) ✓ $x=\frac{5}{2};x=-2$ ✓ answer (2)
9.2	$g'(x)=0$ $-6x^2-6x+12=0$ $x^2+x-2=0$ $(x+2)(x-1)=0$ $x=-2$ or $x=1$ at T: $x=1$	✓ $g'(x)=0$ ✓ $g'(x)=-6x^2-6x+12$ ✓ factorisation ✓ answer (4)
9.3	$g'(x)=-6x^2-6x+12$ $g'(-3)=-6(-3)^2-6(-3)+12$ $g'(-3)=-54+18+12$ $g'(-3)=-24$ $y=ax+q$ $11=-24(-3)+q$ $q=-61$ $y=-24x-61$ **OR** $g'(x)=-6x^2-6x+12$ $g'(-3)=-6(-3)^2-6(-3)+12$ $g'(-3)=-54+18+12$ $g'(-3)=-24$ $y-11=-24(x+3)$ $y-11=-24x-72$ $y=-24x-61$	✓ $g'(-3)$ ✓ -24 ✓ method of setting up straight line equation ✓ substitution of point $(-3\,;11)$ ✓ answer in equation form (5) ✓ $g'(-3)$ ✓ -24 ✓ formula ✓ substitution of point $(-3\,;11)$ ✓ answer in equation form (5)

QUESTION 10

10.1	$V=\pi r^2h$ $200=\pi r^2h$ $h=\frac{200}{\pi r^2}$	✓ formula ✓ substitution (2)
10.2	Surface Area $=2\pi rh+\pi r^2$ $S(r)=\pi r^2+\frac{200}{\pi r^2}.2\pi r$ $S(r)=\pi r^2+\frac{400}{r}$	✓ formula ✓ substitution (2)
10.3	$S(r)=\pi r^2+400r^{-1}$ $\frac{dS}{dr}=2\pi r-400r^{-2}$ At minimum : $\frac{dS}{dr}=0$ $2\pi r-\frac{400}{r^2}=0$ $\pi r^3-200=0$ $r^3=\frac{200}{\pi}$ $r=3{,}99$ cm	✓ exponents correct ✓ $\frac{dS}{dr}=2\pi r-400r^{-2}$ ✓ $\frac{dS}{dr}=0$ ✓ $r^3=\frac{200}{\pi}$ ✓ $r=3{,}99$ or $r=\sqrt[3]{\frac{200}{\pi}}$ (5) Note: If did not put = 0, penalise 1 mark If notation is $\frac{dy}{dx}$, ignore notation. **[9]**

QUESTION 11

11.1	$10x + 8y \le 800$ $3x + 4y \le 360$ $y \ge 60$ $x, y \in N_0$	✓ answer ✓ answer ✓ answer (3)
11.2 & 11.3	See attached graph … (5) See attached graph … (1)	11.2 ✓✓ $y = -\frac{3}{4}x + 90$ ✓✓ $y = -\frac{5}{4}x + 100$ ✓ $y = 60$ (5) 11.3 ✓ feasible region (1) Note: If shading only, and did not state feasible region 1/1
11.4	$P = 200x + 250y$	✓ answer (1)
11.5	$250y = -200x + P$ $y = -\frac{4}{5}x + \frac{P}{250}$ Maximum at (20 ; 75)	✓ gradient ✓ search line ✓ answer (3) Note: Read correctly from the candidate's graph for the point for maximum profit. If used vertices method: 1/3 for accurate answer.
11.6	$m = -\frac{3}{4}$ Since the gradient of the new profit function is equal to the gradient of the constraint $3x + 4y \le 360$, there are points other than (20 ; 75) that give an optimal solution.	✓ $m = -\frac{3}{4}$ ✓✓ more points in optimal solution (more than one solution) (3) Note: If just answer Yes 0 / 3 If just answer No 0 / 3 **[16]**